Good friends Bahaa and Helena are each working on some top-secret government projects. They would like to discuss the projects they both work on, but due to secrecy rules, must not reveal that they are working on a particular project unless their friend is also working on that same project.

Together, the two friends have devised a cooperative game using playing cards to tell them what they need to know. Here is how the game works:

- Bahaa and Helena identify a particular project; let’s call it Project X.
- Each of Bahaa and Helena takes one Ace of Hearts and one Ace of Spades. An additional Ace of Spades is placed face down in front of them. (All cards of the same type should be identical.)
- Bahaa goes first. Bahaa will place his two cards face down on top of the card already on the table. If Bahaa is working on the project, he first puts down the Ace of Spades and then the Ace of Hearts. If Bahaa is not working on the project, then he puts down the Ace of Hearts, then the Ace of Spades.
- Next, it’s Helena’s turn. Helena will place her two cards face down on top of the three cards already on the table. If Helena is working on the project, she first puts down the Ace of Hearts and then the Ace of Spaces. If Helena is not working on the project, she first puts down the Ace of Spades and then the Ace of Hearts.
- Notice that she does the opposite of what Bahaa did.
- Now Bahaa and Helena ask a mutual friend to pick up the deck. While Bahaa and Helena close their eyes, the friend places the cards from the deck face up, in order, so that they form a circle. The friend is not aware of the rules of the game and must place the cards in order, but so that Bahaa and Helena cannot tell which card was on the top of the pile by looking at the circle of cards.
- Looking at the circle of cards, Bahaa and Helena now know what information about the project it is safe to share with one another.

**Activity:** Let’s explore how Bahaa’s and Helena’s game works!

Decide which person will play the role of Bahaa and which player will play the role of Helena. You can enlist a friend to secretly place the cards in a circle for you, but you do not need to. You can also use homemade cards if you do not have three identical decks of playing cards.

Play four different rounds of this game, and complete the table below:

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bahaa on Project X?</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Helena on Project X?</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Resulting card circle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To learn how Bahaa and Helena can use this game to find out which projects they both work on, think about the questions:

1. The four rounds from the activity show what the result of the game would be in all four possible cases for Project X. Three of the four cases result in card circles that can be rotated so that they all look the same. The remaining case results in a card circle that looks different from the other three, no matter how the circle is rotated. Which three cases result in card circles that look the same (when rotated)?

2. If Bahaa is working on Project X, then he can use the card circle to determine whether or not Helena is also working on Project X. Can you explain why?

3. If Bahaa is not working on Project X, then he cannot use the card circle to determine whether or not Helena is working on Project X. Can you explain why?

Try this!

Make up your own table for four different projects.

<table>
<thead>
<tr>
<th>Player’s Name</th>
<th>Project 1</th>
<th>Project 2</th>
<th>Project 3</th>
<th>Project 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player’s Name</td>
<td>Project 1</td>
<td>Project 2</td>
<td>Project 3</td>
<td>Project 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each player should fill in their name, and place a “Yes” or “No” for each project, but keep their answers hidden from the other player. Play the game again and see if you can use what you have learned about the game to determine which projects you are both working on.

Note that the role of the mutual friend can be replaced by the players taking turns “cutting the cards” a few times after they have all been placed in the pile. To cut a pile of cards, you split the pile in two, lifting the upper part of the pile from the top and placing the lower part of the pile on top of it.

Remember the goal of the game is to achieve the following:

- If you are working on a project, then you can use the card circle to determine whether or not the other person is also working on the project, and
- If you are not working on a project, then you cannot use the card circle to determine whether or not the other person is working on the project.

More Info:

Check out the CEMC at Home webpage on Friday, June 19 for a solution to It’s in the Cards.