CEMC at Home

Grade 4/5/6 - Friday, May 22, 2020
Toothpick Polyhedrons - Solution

1. Construct a model of a square-based pyramid. How many toothpicks did you use? How many marshmallows did you use? Is there more than one way to do this?

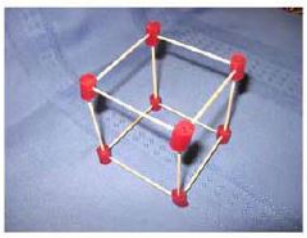
   Solution:

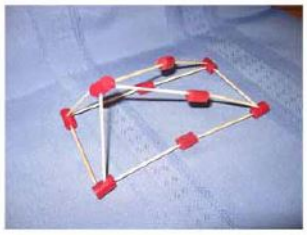
   You can construct a model of a square-based pyramid in more than one way. For example, you can do so using 8 toothpicks and 5 marshmallows, or 12 toothpicks and 9 marshmallows. A diagram of this second way is shown at the end of the solution to 2. below.

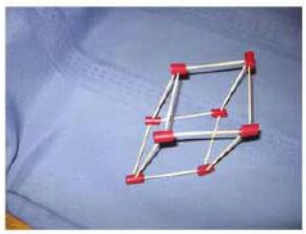
2. See how many different polyhedrons you can construct using exactly 12 toothpicks.

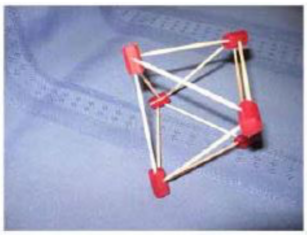
   Solution:

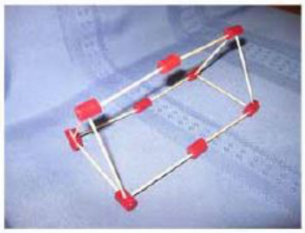
   Here are some photos of models of various polyhedrons which can be built using exactly 12 toothpicks.

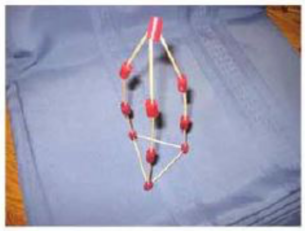

Cube


Rectangular-Based Pyramid


Rhombohedron

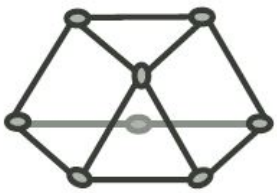

Octahedron

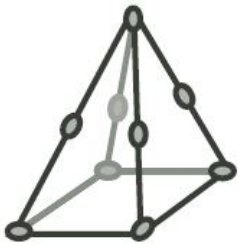

Triangular Prism

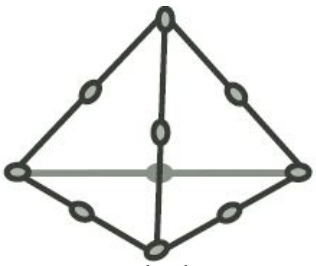

Triangular-Based Pyramid

Notice that the sides of the triangular-based pyramid have sagged a bit under their own weight before the photo was taken. The three toothpicks forming the edges joining at the top vertex should actually be straight.

Here are some sketches of a few more polyhedrons you can model with exactly 12 toothpicks.


You name it!

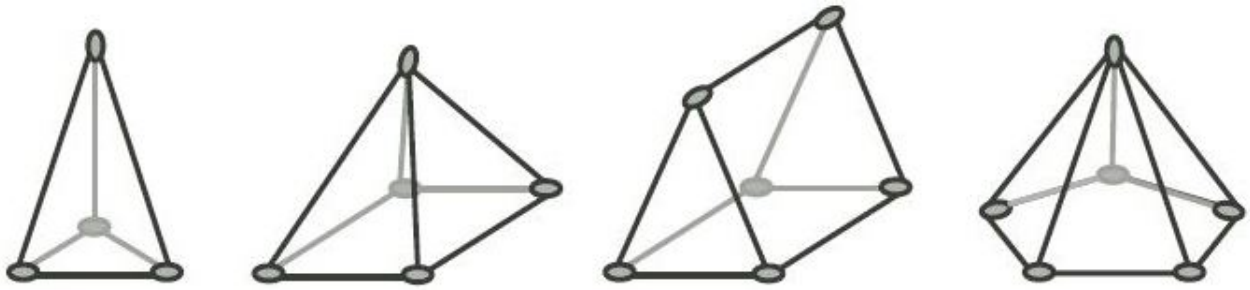

Square-based Pyramid


Tetrahedron

3. Now see how many different polyhedrons you can construct using *fewer* than 12 toothpicks.

   Solution:

   Here are some polyhedrons that you can model using *fewer* than 12 toothpicks. Three of them are pyramids and one is a prism. Can you name each of the polyhedrons?



4. Can you construct a polyhedron with a hexagonal base using exactly 12 toothpicks? Why, or why not?

   Solution:

   It is not possible to do this.

   If you try to construct a model of a polyhedron with a hexagonal base, then the base must use at least 6 toothpicks to construct. This would leave you with at most 6 toothpicks to form the other edges. Each of the 6 vertices of the base must have a toothpick coming out, but when you try joining them to form a "peak", you will discover that you actually get a two-dimensional figure!

   This is because a regular hexagon (with equal sides the length of one toothpick) is formed from six equilateral triangles. So your toothpick model would have the shape of the diagram at the right.