In this activity, we will play a game with X’s and O’s on a rectangular grid.

You Will Need:

- Two players
- Some paper
- A pen or pencil

How to Play:

1. Start by drawing a rectangular grid of squares to use as a game board.
   
   A $3 \times 5$, $4 \times 4$, and $5 \times 5$ game board have been provided for your use on the last page, but you may create game boards of any size.

2. Players will take turns. Decide which player will go first (Player 1) and which player will go second (Player 2).

3. To begin, Player 1 marks any square in the grid with an X.

4. Next, Player 2 marks any other square in the grid with an O, except that they cannot mark any square that is touching the square with the first X.
   
   Two squares are touching if they share a side or a vertex. For example, in the game board shown below, Player 2 may not put an O in any of the squares indicated in grey.

5. From here, the two players take turns marking the grid with X’s (Player 1) and O’s (Player 2). Players can never mark a square that touches a square already marked by either player.

6. The first player who is unable to make a move loses, and the other player wins!

Play this game a number of times.

Try all three of the game boards on the last page, and alternate who goes first and who goes second. Think about the questions on the next page.
Questions:

1. Play the game on the $3 \times 5$ game board. How many moves are there in the shortest possible game? How many moves are there in the longest possible game?

2. Suppose Player 1 makes the first move shown below.

   ![Game Board](image)

   How can Player 2 win the game on their first move?

3. Now suppose Player 1 starts the game by claiming the middle square as shown below.

   ![Game Board](image)

   After making this move, it turns out there is a strategy Player 1 can use to win every time. Can you figure out Player 1’s strategy?

4. Can you adjust the strategy you found in 3. so that it works for a $5 \times 5$ game board?

5. Play the game on the $4 \times 4$ game board. Which player (Player 1 or Player 2) seems to win most often? Can you come up with a strategy that will make sure this player wins the game every time?

6. Play the game on other larger game boards of your choosing. See if you can find strategies for winning each game!

More Info:

Check the CEMC at Home webpage on Friday, May 15 for a discussion of Keeping Your Distance.
Game Boards

[Three diagrams of game boards]