Long ago, there was a farmer whose land was in the shape of a square, each side being exactly 100 metres long.

One day, a woman knocked on the farmer’s door and begged for something to eat. Being a kind person, the farmer fed her a nice lunch.

After the woman had eaten, she said “Kind farmer, I am your Queen! As a reward for your kindness, I will grant you enough additional land that you may double the area of your farm. However, your land must remain in the shape of a square.”

Let’s help the farmer figure out how he might determine the side length of his new square of land.

(a) A first thought might be to simply double the length of each side of the original square of land. That is, the new square of land would have sides that are each 200 metres long.

Draw a diagram of his original square of land and of this proposed new one. Explain why this solution does not give the desired result of a new square of land with twice the area of the original one.

(b) Suppose each square below represents a copy of the farmer’s original square of land, so their combined area represents the area of the new square of land. Can you divide the given squares in such a way that the pieces can be reassembled in order to form a single larger square? Can you convince yourself that the shape you created is, in fact, a square?

*You may cut and arrange these two squares in any way you would like, but your rearrangement must use all of these pieces, be in the shape of a square, and not change the overall area.*

(c) Can you relate the side length of the larger square you created in part (b) to either the side length or the diagonal length of the smaller square?

**Challenge:** If the farmer’s land had been a rectangle, but not necessarily a square, would your technique from part (b) produce a single larger rectangle twice the area of the original rectangle?

**More info:**
Check out the CEMC at Home webpage on Friday, May 8 for a solution to This Farmer is No Square.