Problem:

(a) Using their tree as the key, what is the code for the word **LOST**?

(b) Jacques sent Charlotte the following secret message.
\[ \ast \, \square \, \triangle \, \square \, \triangle \]

Explain why Jacques must have made a mistake when encoding the message.

(c) Using their tree as the key, decode the message displayed below.

\[ \triangle \, \square \, \square \, \ast \, \ast \, \triangle \, \star \, \square \, \ast \, \triangle \, \star \, \square \, \ast \, \square \, \square \, \ast \, \ast \, \triangle \, \star \, \square \, \square \, \ast \, \triangle \, \triangle \]

Solution:

(a) The code for **LOST** is: \[ \ast \, \square \, \ast \, \triangle \, \square \, \triangle \]

(b) Here is what we get when we try to decode this message:

Starting at the top circle, \[ \ast \rightarrow \text{left}, \, \square \rightarrow \text{middle}, \, \triangle \rightarrow \text{right} \Rightarrow \text{L} \]

Going back to the top, \[ \square \rightarrow \text{middle} \Rightarrow \text{E} \]

Going back to the top, \[ \triangle \rightarrow \text{right} \Rightarrow ? \]

We have now run out of symbols and are unable to complete the process. Since \[ \triangle \] alone is not the code for a letter, this message cannot have been encoded correctly.

(c) The decoded message is: **SECRET CODES ARE COOL** (or “secret codes are cool”).

Extension:

1. Draw the tree that matches the codes in the table.

<table>
<thead>
<tr>
<th>A</th>
<th>\ast</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>\square , \star , \star</td>
</tr>
<tr>
<td>D</td>
<td>\triangle , \star</td>
</tr>
<tr>
<td>E</td>
<td>\square , \star , \square</td>
</tr>
<tr>
<td>L</td>
<td>\triangle , \square</td>
</tr>
<tr>
<td>O</td>
<td>\square , \star , \triangle</td>
</tr>
<tr>
<td>R</td>
<td>\square , \square</td>
</tr>
<tr>
<td>S</td>
<td>\square , \square</td>
</tr>
<tr>
<td>T</td>
<td>\triangle , \triangle</td>
</tr>
</tbody>
</table>

2. Charlotte and Jacques think that the tree they used in the problem above is better than the tree that matches the codes that you found in 1. Do you agree with this? Why or why not?
Solution:

1. The tree that matches this encoded table is:

2. Let’s compare the two trees:

When using the tree on the left (created by Charlotte and Jacques) to encode the message

```
SECRETCODESARECOOL
```

the encoded message has 35 symbols. Go back and count them for yourself.

This is already pretty long! However, if we use the new tree on the right to encode the same message, the encoded message will have even more symbols, which means we would have to send a longer message. The encoded message in this case would have 45 symbols in total. Can you see why?

We can determine the number of symbols the encoded message will have by counting how many symbols are needed to code each letter, and how many times each letter appears in the original message.

The reason it takes more symbols to encode the message with the second tree is that the letters that appear most often in the original message (C, E, and O) happen to be the letters with the longest codes (three symbols instead of one or two).

If you look at the first tree, you will see that the letters C, E, and O have codes of length one or two, rather than three.

So it looks like the first tree (created by Charlotte and Jacques) is a better choice for encoding this particular message since it takes fewer symbols to do so. What about for other messages?