Introduction: Charlotte and Jacques want to send secret messages to each other. They will use three different symbols in their messages: ⋆, □, and △.

To encode their secret messages they use a key that only they know. Encoding a message means changing the message from regular text to code involving the symbols ⋆, □, and △.

This same key can be used to decode their secret messages. Decoding a message means changing the message from code involving the symbols ⋆, □, and △ back to regular text.

The key they use to encode and decode messages is the tree shown below.

Notice that each of the letters A, B, C, D, E, F, G, and H appear as leaves on this tree. This means that this tree can be used to encode any message that contains only these letters.

Each letter has its own code that is determined by the letter’s placement on the tree. The code for a letter can be found by following the path on the tree from the top circle to the circle containing the letter, and picking up all of the symbols on the branches that you travel along on this path.

Notice that each left branch has the symbol ⋆, each middle branch has the symbol □, and each right branch has the symbol △.

For example, the code for the letter C is □ ⋆ △.

We find this code by starting at the top circle and taking a middle branch (picking up a □), followed by a left branch (picking up a ⋆), followed by a right branch (picking up a △), to end up at C.

As another example, the code for the letter G is △ □.

We find this code by starting at the top circle and taking a right branch (picking up a △), followed by a middle branch (picking up a □), to end up at G.

To encode a message, we replace each letter in the message with its code from the tree. For example, the encoded version of BAG is □ ⋆ ⋆ ⋆ △ □ as shown below.

We have already seen the code for the letter G, but you should use the tree to check that these are the correct codes for B and A. Notice that codes for different letters can be of different lengths.
Example: A message was encoded using the given tree and the resulting code is shown below. Use the tree to decode the message.

□ ★ △ ★ □ □ △

Explanation: You can decode this message letter by letter. Start at the top circle of the tree and follow the path indicated by the symbols until you end up at a letter.

To follow the path indicated by these symbols you must follow a middle branch (□), then a left branch (★), then a right branch (△), and this takes you to the letter C. Once you reach a letter you stop, and restart the process at the top circle of the tree with the next part of the sequence.

To follow the path indicated by the remaining symbols ★ △ □ □, we must follow a left branch (★) which leads us straight to the letter A, and so we stop and reset again. Continuing in this way until we reach the end of the code, we reveal the original message of CAGE as shown below:

Starting at the top circle, □ → middle, ★ → left, △ → right ⇒ C
Going back to the top, ★ → left ⇒ A
Going back to the top again, △ → right, □ → middle ⇒ G
Going back to the top again, □ → middle, △ → right ⇒ E

Problem: It is always a good idea to change your key often to keep your messages safe. To send their messages tomorrow, Charlotte and Jacques will use the following different tree.

(a) Using their tree as the key, what is the code for the word LOST?

(b) Jacques sent Charlotte the following secret message.

★ □ △ □ △

Explain why Jacques must have made a mistake when encoding the message.

(c) Using their tree as the key, decode the message displayed below.

△ □ □ ★ △ ★ □ △ △ ★ ★ △ ★ ★ △ ★ □ □ □ ★ ★ △ ★ ★ △ ★ ★ △ ★ □ □ △

Extension:

There are different trees that will encode exactly the same letters, but in a different way. Suppose that the codes for the letters in a tree are as shown in the table on the right.

1. Draw the tree that matches the codes in the table.

2. Charlotte and Jacques think that the tree they used in the problem above is better than the tree that matches the codes that you found in 1. Do you agree with this? Why or why not?

More Info:

Check out the CEMC at Home webpage on Wednesday, May 6 for a solution to Cool Codes.