Problem 1

(a) The perimeter consists of two lengths $\ell$ and two widths $w$. Since adding two widths and two lengths together gives 16 cm, adding one width and one length together must give $16 \div 2 = 8$ cm. This means the sum of the length and the width must be 8 cm, or

$$\ell + w = 8 \text{ cm}$$

(b) There are only three ways to make 8 by adding two different positive whole numbers:

$$1 + 7 = 8, \ 2 + 6 = 8, \ 3 + 5 = 8$$

Since the length is greater than the width, the only possibilities for $\ell$ and $w$, in centimetres, are shown in the table below.

<table>
<thead>
<tr>
<th>width $w$</th>
<th>length $\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Problem 2

(a) With the sides labelled as shown in the top diagram on the right, we see that the length $\ell$ is equal to three widths $w$.

(b) The only pair from Problem 1(b) for which the length is three times the width is the pair $\ell = 6$ cm and $w = 2$ cm.

(c) If each smaller identical rectangle has length 6 cm and width 2 cm, then the area of each smaller rectangle is $6 \times 2 = 12 \text{ cm}^2$.

Since there are 5 smaller rectangles making up the larger rectangle, the area of the larger rectangle must be $5 \times 12 \text{ cm}^2 = 60 \text{ cm}^2$.

Challenge Problem

(a) Since the total area of the larger rectangle is 84 cm$^2$ and it is formed using seven smaller identical rectangles, the area of each smaller rectangle must be $84 \div 7 = 12 \text{ cm}^2$.

(b) With the sides labelled as shown in the top diagram on the right, we see that three lengths $a$ are equal to four widths $b$. Since the area of each smaller rectangle is 12 cm$^2$ we know that $a$ times $b$ must be 12. The factor pairs of 12 are 1 and 12, 2 and 6, and 3 and 4. The only pair that satisfies the correct relationship is 3 and 4. This means $a = 4$ cm and $b = 3$ cm.

(c) Using the labelled diagram, we see that the larger rectangle has length 12 cm and width 7 cm. This means its perimeter is

$$12 \text{ cm} + 7 \text{ cm} + 12 \text{ cm} + 7 \text{ cm} = 38 \text{ cm}$$