Jessie’s Jewelry

Introduction: Jessie likes to make bracelets and necklaces using four different types of coloured beads.

Although the patterns of the beads in the jewelry may look random, Jessie actually follows strict rules to make the jewelry. Each rule mentions shapes and beads, and explains how to replace a certain shape with a new pattern of shapes and beads.

For example, one of Jessie’s replacement rules for making jewelry is shown below.

The symbols \( \bigstar \) and \( \bigtriangleup \) in the rule represent placeholder shapes.

The symbols \( Y \) and \( G \) represent two of the four different types of beads.

The rule shown above says that whenever Jessie sees the shape \( \bigstar \) in a pattern, this shape can be replaced with one of two different things:

- the pattern \( Y \bigtriangleup Y \), or
- the single bead \( G \).

When making a new piece of jewelry, Jessie always starts out with the single placeholder shape \( \bigtriangleup \) and then uses the three replacement rules shown below.

Jessie applies these rules until a final pattern consisting of only beads is reached.
Example: Let’s see Jessie’s rules in action in an example. Here is an outline of one of Jessie’s attempts to create a simple piece of jewelry.

Notice how Jessie started with the shape △ and ended with a pattern of only beads.

See the table below for an explanation of how this final pattern of beads was created using the rules.

<table>
<thead>
<tr>
<th>Current Stage of Pattern</th>
<th>Rule Applied</th>
<th>Next Stage of Pattern</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>△ → △ ∈ □</td>
<td>□</td>
<td>Jessie had two choices for what pattern to substitute for the shape △ and chose the second option.</td>
</tr>
<tr>
<td>□</td>
<td>□ → □ ∈ ♦</td>
<td>♦</td>
<td>Jessie had two choices for what pattern to substitute for the shape □ and chose the first option.</td>
</tr>
<tr>
<td>♦ ♦</td>
<td>♦ → ♦ ∈ ♦</td>
<td>♦</td>
<td>Jessie had many choices here. Jessie could replace the ♦ with one of two patterns and/or replace the △ with one of two patterns. Jessie chose to apply the rule for ♦ only and chose the second pattern option this time.</td>
</tr>
<tr>
<td>♦ ♦</td>
<td>♦ → ♦ ∈ ♦</td>
<td>♦</td>
<td>Jessie had two choices for what pattern to substitute for each of the ♦ shapes and chose the second option for each of the two ♦ shapes.</td>
</tr>
</tbody>
</table>

Problems:

1. Start with a △ shape and work through the rules yourself. Make different choices than Jessie did in the example above, and create a different pattern of beads than the one shown above.

2. Explain how Jessie can start with a △ shape and apply the rules to end up with the following final pattern of beads:

   ![Pattern](G R G R G)

3. No matter how Jessie chooses to apply the rules, there is no way to create the final pattern of beads shown below by starting with a △ shape. Can you explain why Jessie cannot create this final pattern?

   ![Pattern](G R V G Y G Y)

More Info:

Check out the CEMC at Home webpage on Wednesday, April 29 for a solution to Jessie’s Jewelry.

See the next page for an extra challenge problem to try!
Extra Challenge: Which of the following patterns of beads can be made according to Jessie’s rules? Explain your answers.

Y G Y

G V G R G V G

Y G R G Y V Y G R G Y