Play the game a number of times and think about the following questions.

1. What is the least possible ones digit of the sum that you could get on your turn? What is the greatest possible ones digit you could get?
   
   Solution: The least possible ones digit is 0. This comes from a sum of 10, which happens if you roll a pair of fives, or a six on one die and a four on the other.
   
   The greatest possible ones digit is 9. This comes from a sum of 9, which happens if you roll a six on one die and a three on the other, or a five on one die and a four on another.

2. If you get a small ones digit, in which square should you place it? Why?
   
   Solution: For the four-digit number, the four squares from left to right are the thousands, hundreds, tens, and ones digit, respectively. Since the winner is the player with the greatest four-digit number, small digits should be placed as far as possible to the right. For example, if a digit of 1 comes up, it would make sense to place the digit 1 in an empty square that is farthest to the right.
   
   How should you deal with a ones digit of 0 if it comes up? How should you deal with numbers that are smaller than average, but not as small as 1 (for example 3 or 4)?

3. If you get a large ones digit, in which square should you place it? Why?
   
   Solution: Large digits should be placed as far as possible to the left. For example, if a digit of 9 comes up, it would make sense to place the digit 9 in an empty square that is farthest to the left.
   
   How should you deal with digits that are larger than average, but not as large as 9? If a ones digit of 6 or 7 comes up first, then where should you place it? You might decide to place the digit 7 in the leftmost square, and then have a ones digit of 9 come up next!

4. Which are the least likely ones digits to occur in this game? Which are the most likely ones digits to occur in this game?
   
   Solution: Label the two dice “Die #1” and “Die #2”. There are 36 possible possible rolls and the chances of getting each of these rolls is the same. Below is a table showing the sum of the two dice in each of the 36 possible rolls.

   | Die #2 |   |   |   |   |
---|-----|---|---|---|---|
| 1   | 2  | 3  | 4  | 5  | 6  | 7  |
| 2   | 3  | 4  | 5  | 6  | 7  | 8  |
| 3   | 4  | 5  | 6  | 7  | 8  | 9  |
| 4   | 5  | 6  | 7  | 8  | 9  | 10 |
| 5   | 6  | 7  | 8  | 9  | 10 | 11 |
| 6   | 7  | 8  | 9  | 10 | 11 | 12 |
Notice that there are two rolls which yield a sum with a units digit of 1: a six on Die #1 and a 5 on Die #2, or a five on Die #1 and a 6 on Die #2, giving $6 + 5 = 5 + 6 = 11$. Similarly, a ones digit of 2 can happen in two ways, $1 + 1 = 2$ or $6 + 6 = 12$, and a ones digit of 3 can happen in two ways, $1 + 2 = 3$ or $2 + 1 = 3$. We can see from the table that the ones digits of 1, 2, and 3 are the least likely to occur in the game (with two ways to achieve each digit). Similar reasoning shows that there are

- three ways to get a ones digit of 4 ($2 + 2 = 4$, $1 + 3 = 4$, $3 + 1 = 4$),
- three ways to get a ones digit of 0 ($5 + 5 = 10$, $6 + 4 = 10$, $4 + 6 = 10$),
- four ways to get a ones digit of 5 ($1 + 4$, $4 + 1$, $2 + 3$, $3 + 2$),
- four ways to get a ones digit of 9 ($3 + 6$, $6 + 3$, $4 + 5$, $5 + 4$),
- five ways to get a ones digit of 6 ($1 + 5$, $5 + 1$, $2 + 4$, $4 + 2$, $3 + 3$),
- five ways to get a ones digit of 8 ($2 + 6$, $6 + 2$, $3 + 5$, $5 + 3$, $4 + 4$), and
- six ways to get a ones digit of 7 ($1 + 6$, $6 + 1$, $2 + 5$, $5 + 2$, $3 + 4$, $4 + 3$).

Thus the most likely ones digit is 7 (with six ways to achieve this digit).

**Variations:**

A. Try the same game but with three squares, and then with five squares.

*These games will be similar, but will end after a different number of rolls.*

B. Try the same game but take the ones digit of the product of the two numbers rolled on the dice, instead of the sum. How does this change the answers to the earlier questions?

*The chances of getting certain ones digits change in this variation. For example, the most likely ones digits to occur in this version are 0, 2, and 6, and the least likely ones digit to occur is 7, as it cannot occur at all! Otherwise, the basic strategy of the game remains the same.*