CEMC at Home

Grade 11/12 - Wednesday, May 27, 2020
Triangle of Integers

Here we revisit the idea of using computer science to solve previous CEMC math contest problems.

First, read Question 21 from the 2011 Fermat Contest and do your best to determine the correct answer. Read the solution to this question, and then answer the following related questions.

Problem Set 1

Suppose integers are arranged in a triangle as described in Question 21 of the 2011 Fermat Contest. Further, we will number the rows from top to bottom starting from 1 at the top.

1. What is the number of the row that contains the number 2020?
2. What is the sum of the numbers in the row immediately below the row that contains the number 500?

We will now look at a Python computer program that has been built to help you test your solutions to the questions above, and also to solve further related questions.

Here are instructions for using the program:

1. Open this webpage in one tab of your internet browser. You should see Python code.
2. Open the free CS Circles console in another tab.
3. Copy the code and paste it into the console of the interpreter.
4. Hit *Run program*.
5. You should see that the program outputs the correct answer to Question 21 of the 2011 Fermat Contest.

Try to understand how the program computes and displays the correct answer to the question. It is okay if you are new to computer programming or the language Python! The code is also included below for convenience and the notes below outline some of the details.

Python program

When a Python program is run, anything beginning with a `#` character through to the end of the line is a *comment* which is ignored when the program is run.

When the `while` *loop* is reached, Python tests if the *condition* `n*(n+1) <= 2*num` is true. If it is true, the indented line `n = n + 1` is executed. This process is repeated until a test determines that the condition is false in which case the program continues after the indented line.

The line `n = n + 1` increases the value of the variable `n` by 1. For example, if the value of `n` is 4 when this line is reached, then after this line is executed, the value of `n` becomes $4 + 1 = 5$.

The two characters `//` form an *operator* used to divide two integers.

```
num = 400

# set n to be the number of the row containing num
n = 1
while n*(n+1) <= 2*num:
    n = n + 1

# compute the last numbers in rows n-1 and n
a = ((n-1)*n) // 2
b = (n*(n+1)) // 2

# add up the numbers in row n and display this sum
total = 0
for i in range(a+1,b+1):
    total = total + i
print(total)
```

Do you see where the formulas for a and b above come from? See the solution to the contest problem.

Revisiting Problem Set 1

Let's revisit the questions on the previous page, and try to answer them using elements of our Python program. Suppose integers are arranged in a triangle as described in Question 21 of the 2011 Fermat Contest. Further, we will number the rows from top to bottom starting from 1 at the top.

1. What is the number of the row that contains the number 2020?

   To answer this question using the Python program, change the line `num = 400` to `num = 2020`, remove the lines after the while loop and add the line `print(n)` at the end. Your code should now look like the code in Program 1 below. Run the program. Does the answer given by this program agree with the answer you had calculated? Can you see why this program produces the desired output?

2. What is the sum of the numbers in the row immediately below the row that contains the number 500?

   Use Program 2 below to answer this question. Can you see what changes have been made to the original Python program to obtain Program 2? Run the program. Does the answer given by this program agree with the answer you had calculated? Can you see why this program produces the desired output?

Program 1

```
num = 2020
n = 1
while n*(n+1) <= 2*num:
    n = n + 1
print(n)
```

Program 2

```
num = 500
n = 1
while n*(n+1) <= 2*num:
    n = n + 1
n = n + 1
a = ((n-1)*n) // 2
b = (n*(n+1)) // 2
total = 0
for i in range(a+1,b+1):
    total = total + i
print(total)
```

Problem Set 2

Now, it's your turn! A solution to each of the following problems can be found that uses elements of the given Python program. Modify the given program, or use parts of it, to answer the questions below about the integers arranged in a triangle as described in the Fermat Contest question. Of course, you should feel free to use any other features that you know or learn about. The correct answers are provided so you can test your programs.

Of course, you can try to solve these problems by hand as well!

1. The sum of the numbers of a row is 34481. What is the number of this row?
   Answer: 41

2. For how many rows is the sum of the numbers in the row between 50000 and 90000?
   Answer: 10

More Info:

Check out the CEMC at Home webpage on Wednesday, June 3 for a solution to Triangle of Integers.