The Travelling Salesperson Problem (also known as the Travelling Salesman Problem or TSP) is a famous problem in mathematics and computer science. It is widely known because it has applications to many problems that affect our everyday lives and has instances that have remained unsolved for years! Here is one way to state the problem:

A salesperson wants to visit $n$ cities. What is the shortest route that the salesperson can take in order to visit each of the $n$ cities exactly once and return to their starting point?

For each pair of cities, the distance between the cities is known. There are many different ways to measure this distance for the purposes of this problem. For example, this distance could be the length of a winding road joining the two cities. Alternatively, it could be the length of the straight line segment joining the two cities. This second option is the measurement of distance that we will use in this activity.

You Will Need:

- Two or more players
- A rectangular pegboard

  *We use a board which is 24 cm wide and 30 cm long. See below for alternatives to the pegboard.*

- 12 pegs
- A string of length 1 m
- A ruler or measuring tape

*If you don’t have a pegboard and pegs at home, you can make something similar for yourself. Some good options might include cardboard with pins, foam with toothpicks, or wood with screws. Be as creative as you like.*
Set Up

We will play a game using the pegboard, the pegs and the string. Here we explain how to setup for the game.

- Attach one of the ends of the string to one of the pegs. You can glue the string to the peg or tie the string around the peg.
- Place the peg with the string attached in the bottom left corner of the board.

*During the game, the players will place the remaining 11 pegs in various places on the board. A possible arrangement is shown in the figure.*

How To Play:

1. Start with the pegboard as outlined in the set up.
2. To start the game, the players first need to place the remaining 11 pegs into the board. The pegs can be placed anywhere on the board. You can decide whether to have one player place all of the pegs, or have the players take turns placing the pegs until all pegs are in place.
3. Players alternate turns working with the board. Decide which player will go first.
4. The first player does the following:
   - Arrange the string so that it touches each peg exactly once and returns to the bottom left peg. Make sure the string is pulled tight. *An example of this is shown below.*
   - Place a finger to mark the place on the string where the string meets itself at the bottom left peg.
   - Unravel the string, making sure to keep the correct mark on the string. Measure the length of the string between the attached peg and the place marked by the finger. Record this measurement as the score for the first player’s first turn.
5. Now all other players take a turn doing what the first player did. All players are trying to find a route for the string that touches each peg exactly once and returns to the bottom left peg, *using the smallest possible length of string.*
6. The game ends after each player has had three turns with the string. The winner of the game is the player who achieved the *smallest* score (measurement) on a single turn.

When you are finished a game, you can pull out all but the bottom left peg and try again! New placements of the pegs will lead to whole new games.
Revisiting the Travelling Salesperson Problem (TSP)

The possible routes players can make with the string in this game model the possible routes in particular instances of the TSP with 12 cities. Each time you set up the peg board, with new distances between pairs of pegs, you are setting up a new version of the TSP, with new distances between the pairs of cities.

It is important to note that winning the game does not mean that you have solved the instance of the TSP associated with this particular board. To win the game, you just need to have the shortest route out of all the routes found in the game. But there may be a shorter route that no player found during the game! Think about how you might convince yourself that you have actually found the shortest route of all possible routes. In general this is hard to do, and gets even harder the more pegs you add to your game (or cities you add to your problem).

Think about the following questions:

- How many different possible routes are there to choose from in each round of the game using 12 pegs?
- If you change the game to include more pegs, say $n$ pegs in total, how many different possible routes are there to choose from in each round of this game?

The TSP has been studied for many decades, yet there is no known efficient algorithm to solve this problem in general. We encourage you to look into this problem more on your own.

More Info:

The TSP can be modelled using graphs. Check out this resource about the TSP to learn more!