We are going to call the diagonal line of white squares indicated in diagram, the \textit{main diagonal}. Playing this game, you probably realized that the main diagonal is important to the strategy of this game.

The rook begins on the main diagonal. The first player moves the rook and no matter what move they make, they will have to move the rook off of the main diagonal. If the first player moves the rook \( n \) squares to the right, then the second player can move the rook \( n \) squares up and the rook will be back on the main diagonal. If the first player moves the rook \( n \) squares up, then the second player can move the rook \( n \) squares to the right and the rook will be back on the main diagonal. In such a way the second player can guarantee that the rook will be on the main diagonal after their turn and the rook will be closer to the top right square (and maybe even at this square)!

Since the rook is back on the main diagonal, the first player must again move the rook off of the main diagonal and the second player can again put it back on to the main diagonal. Repeating this process, the second player will always be able to place the rook on the main diagonal closer to the top right square. Since there are a finite number of squares on the chessboard, the second player will eventually place the rook in the square at the top right corner.

Thus, we can see that the second player has a winning strategy for this game.

\textbf{Variation:}

In the variation of this game, we have a board with only five rows. We refer to the diagonal shown as the main diagonal. In this variation, the rook does not start on the main diagonal. If the first player moves the rook three spaces to the right, the rook will then be on the main diagonal. After this first move, the second player has no choice but to move the rook off of the main diagonal, leaving the first player the opportunity to place it back on the diagonal. Then the strategy continues as described for the first version. Therefore, the first player has the winning strategy in this variation of the game.

\textbf{Extension:} Consider a chessboard with any number of rows and any number of columns. For what size of chessboard will the first player have a winning strategy? For what size of chessboard will the second player have a winning strategy?