CEMC at Home

Grade 7/8 - Thursday, May 21, 2020
Contest Day 3

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

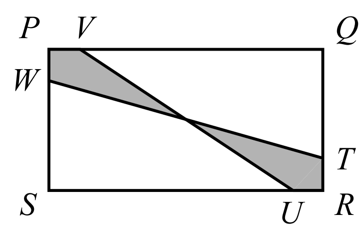
2011 Gauss Contest, #13

Five children had dinner. Chris ate more than Max. Brandon ate less than Kayla. Kayla ate less than Max but more than Tanya. Which child ate the second most?

(A) Brandon **(B)** Chris **(C)** Kayla **(D)** Max **(E)** Tanya

2020 Gauss Contest, #23

In the diagram, rectangle $PQRS$ has $PS = 2$ and $PQ = 4$. Points T, U, V, W are positioned so that $RT = RU = PW = PV = a$. If VU and WT pass through the centre of the rectangle, for what value of a is the shaded region $\frac{1}{8}$ the area of $PQRS$?



(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{5}$

(D) $\frac{1}{3}$ **(E)** $\frac{1}{4}$

More Info:

Check out the CEMC at Home webpage on Monday, May 25 for solutions to the Contest Day 3 problems.

CEMC at Home

Grade 7/8 - Thursday, May 21, 2020
Contest Day 3 - Solution

Solutions to the two contest problems are provided below.

2011 Gauss Contest, #13

Five children had dinner. Chris ate more than Max. Brandon ate less than Kayla. Kayla ate less than Max but more than Tanya. Which child ate the second most?

(A) Brandon **(B)** Chris **(C)** Kayla **(D)** Max **(E)** Tanya

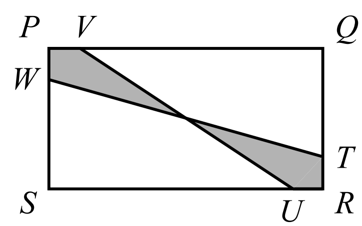
Solution:

Since Kayla ate less than Max and Chris ate more than Max, then Kayla ate less than Max who ate less than Chris.
Brandon and Tanya both ate less than Kayla.
Therefore, Max ate the second most.

ANSWER: (D)

2020 Gauss Contest, #23

In the diagram, rectangle $PQRS$ has $PS = 2$ and $PQ = 4$. Points T, U, V, W are positioned so that $RT = RU = PW = PV = a$. If VU and WT pass through the centre of the rectangle, for what value of a is the shaded region $\frac{1}{8}$ the area of $PQRS$?

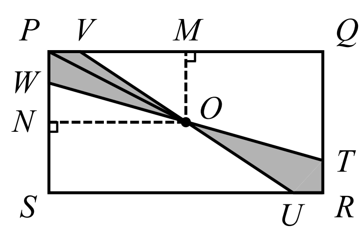


(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{5}$

(D) $\frac{1}{3}$ **(E)** $\frac{1}{4}$

Solution:

We begin by joining the centre of the rectangle, O, to vertex P.
We also draw OM perpendicular to side PQ and ON perpendicular to side PS.
Since O is the centre of the rectangle, then M is the midpoint of side PQ and so $PM = \frac{1}{2} \times 4 = 2$.
Similarly, N is the midpoint of PS and so $PN = \frac{1}{2} \times 2 = 1$.



$\triangle PVO$ has base $PV = a$ and height $OM = 1$, and so has area $\frac{1}{2} \times a \times 1 = \frac{1}{2}a$.
$\triangle PWO$ has base $PW = a$ and height $ON = 2$, and so has area $\frac{1}{2} \times a \times 2 = a$.
Thus, quadrilateral $PWOV$ has area equal to the sum of the areas of these two triangles, or $\frac{1}{2}a + a = \frac{3}{2}a$.
Similarly, we can show that quadrilateral $RTOU$ also has area $\frac{3}{2}a$ and so the total area of the shaded region is $2 \times \frac{3}{2}a = 3a$.
The area of rectangle $PQRS$ is $4 \times 2 = 8$ and since the area of the shaded region is $\frac{1}{8}$ the area of $PQRS$, then $3a = \frac{1}{8} \times 8$ or $3a = 1$ and so $a = \frac{1}{3}$.

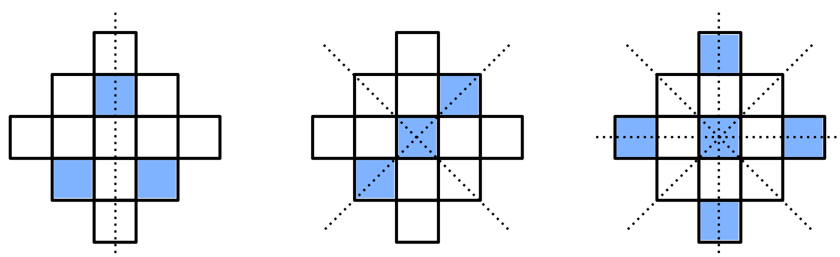
ANSWER: (D)

CEMC at Home

Grade 7/8 - Friday, May 22, 2020
Mirror, Mirror

Thirteen identical squares are arranged as shown in the figure below on the left. In today's activity, we will create symmetrical designs by shading in some of the squares in the figure. Notice that the design with no shaded squares has exactly four lines of symmetry, as shown in the image below on the right.

In each design below, some of the squares in the figure have been shaded. All lines of symmetry for each design are shown. Notice that the rightmost design maintains all four original lines of symmetry, but the other two designs only have one or two lines of symmetry.



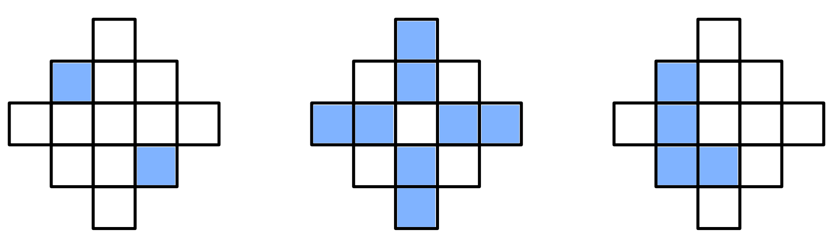
Problem 1: How many different designs are there with exactly two shaded squares that have exactly two lines of symmetry?

You can use the blank squares on the next page to draw some designs.

Problem 2: Is it possible to create a design that has exactly three lines of symmetry? If so, draw one. If not, explain why this is not possible.

You can shade as many squares in the figure as you would like.

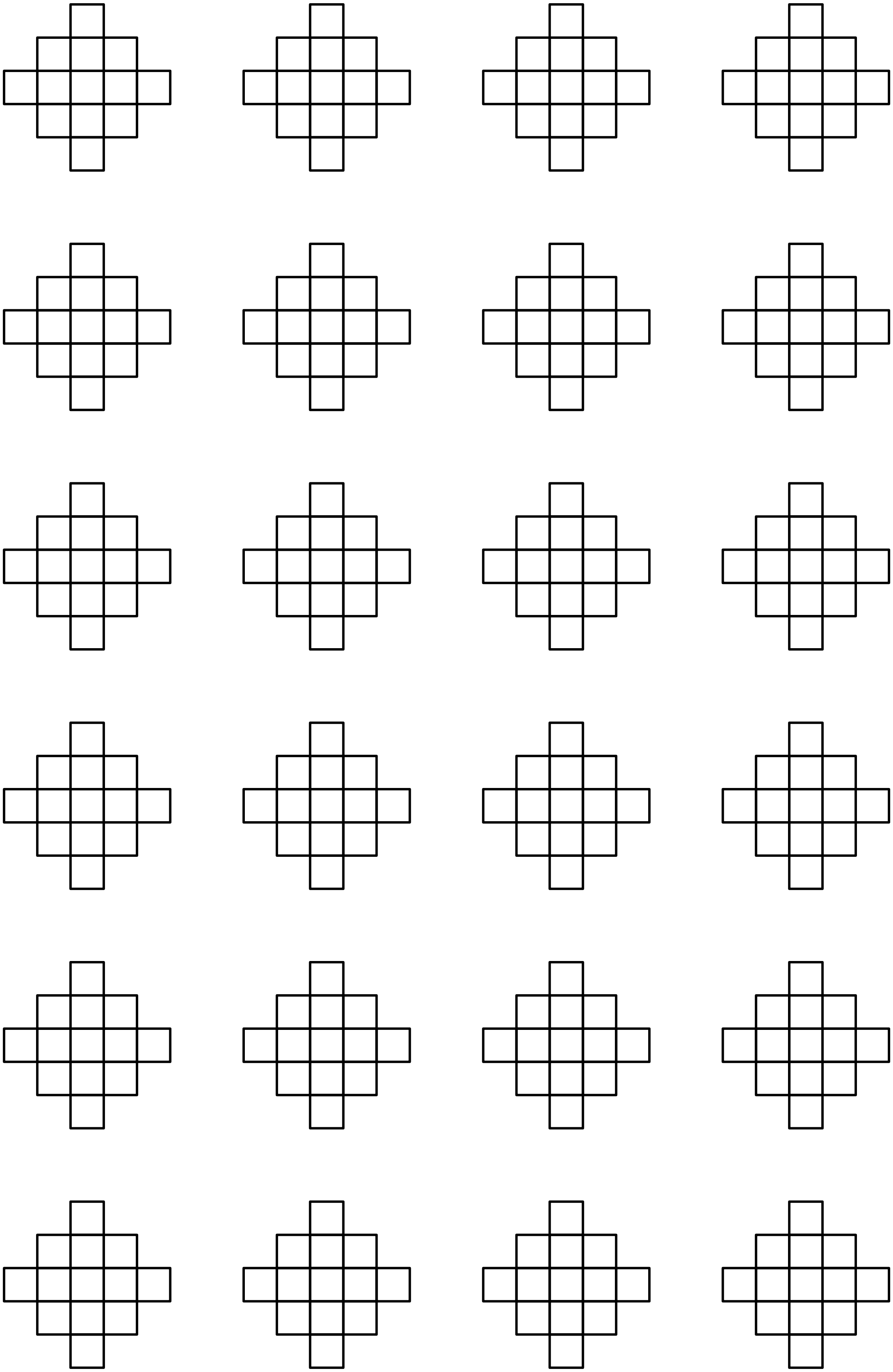
Problem 3: A design has rotational symmetry if we can rotate it about its centre less than a full turn and produce a design that looks identical to the original design. The first two designs below have rotational symmetry but the last design does not.



How many different designs are there with exactly three shaded squares that have rotational symmetry and at least one line of symmetry?

More Info:

Check out the CEMC at Home webpage on Monday, May 25 for a solution to Mirror, Mirror.



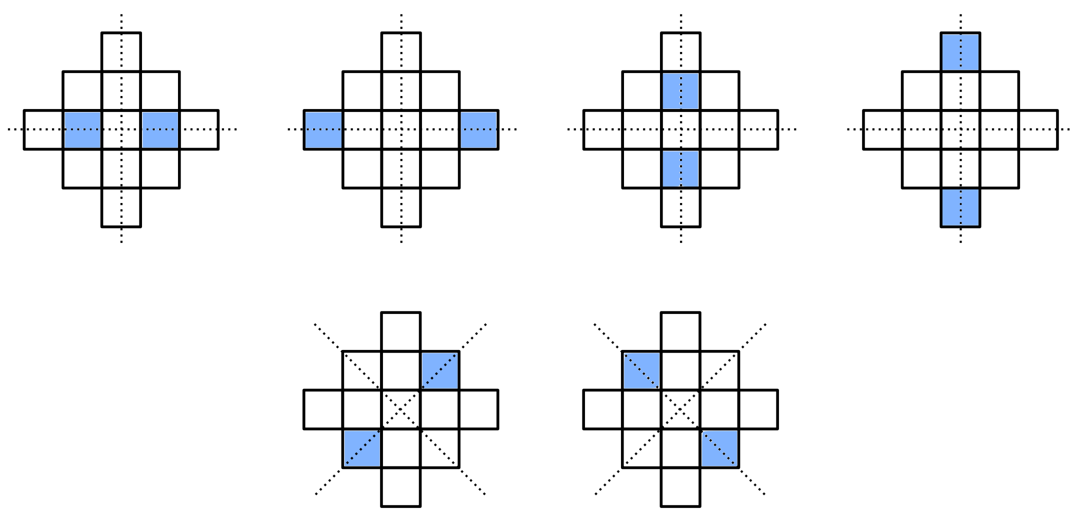
CEMC at Home

Grade 7/8 - Friday, May 22, 2020
Mirror, Mirror - Solution

Thirteen identical squares are arranged as shown in the figure below on the left. Notice that the design with no shaded squares has exactly four lines of symmetry, as shown in the image below on the right.

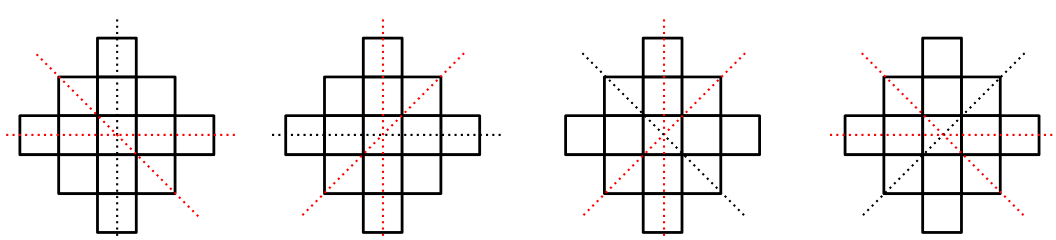
Problem 1: How many different designs are there with exactly two shaded squares that have exactly two lines of symmetry?

Solution: There are six different designs. They are shown below.



Problem 2: Is it possible to create a design that has exactly three lines of symmetry? If so, draw one. If not, explain why this is not possible.

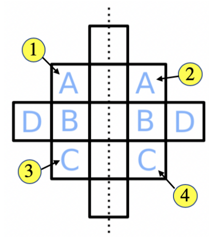
Solution: It is not possible. First, we notice that any group of three of the four lines of symmetry must include a diagonal line *and* a horizontal or vertical line. The four possible combinations of three lines of symmetry are shown below.



Group 1 Group 2 Group 3 Group 4

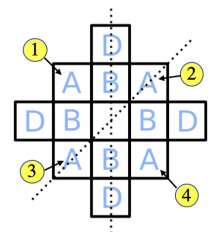
It turns out that any design that has a diagonal line of symmetry *and* a vertical or horizontal line of symmetry must actually have all four lines of symmetry. (This means it cannot be possible to create a design with exactly three lines of symmetry.)

Suppose we have a design that has the three lines of symmetry in Group 2. We will use letters to show the squares that must have the same shading, based on the lines of symmetry. For example, the squares marked with the letter "A" must be either all shaded, or all not shaded.



Since the design has the vertical line of symmetry, the design must have the symmetry indicated in the top image on the right.

Since the design *also* has the diagonal line of symmetry from the lower left corner to the upper right corner, the design must have the additional symmetry shown in the bottom image on the right.

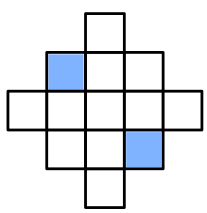
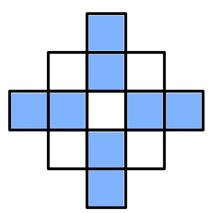
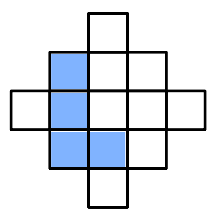


For example, the vertical line of symmetry tells us that the squares marked with 1 and 2 must look the same and that the squares marked with 3 and 4 must look the same. The diagonal line of symmetry tells us that the squares marked with 1 and 4 must look the same. Putting this together, we see that all four of these squares must look the same and so are marked with the same letter, A.

From this, we can see that the design actually has all four lines of symmetry.

This means any design with these two lines of symmetry will actually have all four lines of symmetry. This argument works for Group 2 and Group 3. The argument for Group 1 and Group 4 is similar.

Problem 3: A design has rotational symmetry if we can rotate it about its centre less than a full turn and produce a design that looks identical to the original design. The first two designs below have rotational symmetry but the last design does not.

How many different designs are there with exactly three shaded squares that have rotational symmetry and at least one line of symmetry?

Solution: There are six different designs. They are shown below.