CEMC at Home

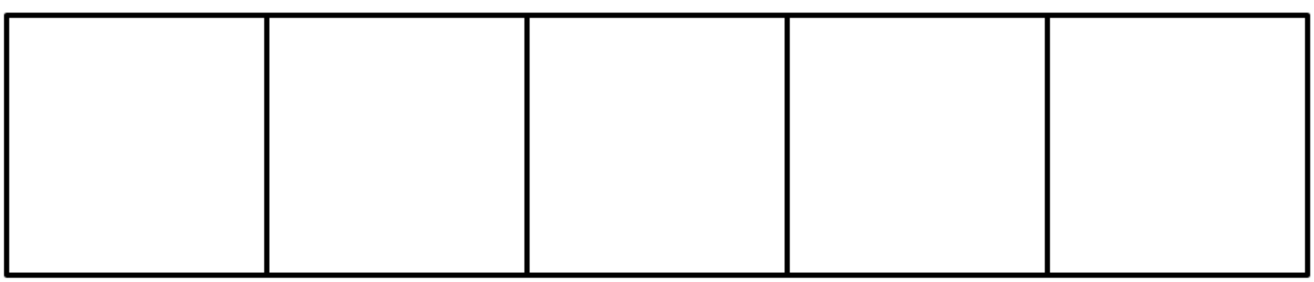
Grade 7/8 - Monday, April 6, 2020
Five Square

You Will Need:

- Two players
- The grid below
- Four matching game pieces for each player (coins, paper clips, etc.)

How to Play:

1. Players alternate turns. Decide which player will go first.
2. On each turn, a player can either
    - put one of their game pieces on an empty square, or
    - put two of their game pieces on two empty squares, as long as the two empty squares are right next to each other.
3. The player to put a game piece on the last empty square wins the game!



Play this game a number of times. Can you come up with a strategy that will allow you to win most of the time? What about every time? Is it better to go first or second or does this not matter?

Variation: Add another square so that the game grid now has six squares. Keep the rules of the game the same. How does this change your strategy for the game?

More Info:

Check the CEMC at Home webpage on Tuesday, April 7 for a discussion of a strategy for Five Square.

CEMC at Home

Grade 7/8 - Monday, April 6, 2020
Five Square - Solution

Hopefully you thought about a strategy for this game while you played the game a number of times. Maybe you found a strategy that helped you win the game more often than you lost. Maybe you found a strategy that helped you win every time. It turns out that this game has what is called a *winning strategy*. This is a strategy that allows you to always win, regardless of what the other player does! Let's talk about a strategy.

Winning Strategy

We will play first and start by placing one game piece on the middle square (square 3).

1	2	3 X	4	5

Then our opponent has two different options for the type of move they play on their turn.

Option 1: Place two game pieces on two empty squares that are right beside each other.

Option 2: Place one game piece on an empty square.

We need to explain how we will win the game no matter what they choose to do on their next turn.

If they choose Option 1, then they will have to place game pieces either on squares 1 and 2 or on squares 3 and 4. If they place game pieces on squares 1 and 2, then on our next move, we will place game pieces on squares 3 and 4 to win the game. If they place game pieces on squares 3 and 4, then on our next move, we will place game pieces on squares 1 and 2 to win the game.

So to have a chance to win, it seems our opponent must go with Option 2 instead of Option 1.

If they choose Option 2, then they will have to place one game piece on one of squares 1, 2, 4, or 5. In this case, on our next move, we will place one game piece on the "other side" of the grid: If they place a game piece on square 1 or 2, then we place one on square 4 or 5, and if they place a game piece on square 4 or 5, then we place one on square 1 or 2. Either way, after these turns, the grid will have exactly two empty squares left, but they will not be right beside each other. Since it is our opponent's turn, they will have to place one game piece only, leaving us the opportunity to fill he grid to win the game!

From this explanation, we see that no matter what our opponent does, we can follow this plan and guarantee a win each game.

Exploring Other Strategies

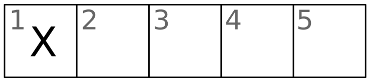
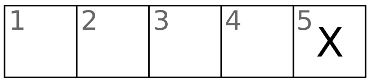
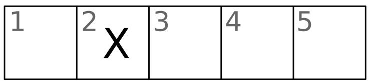
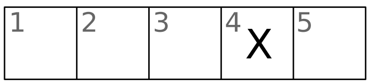
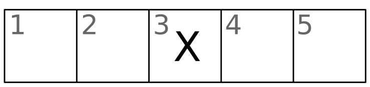
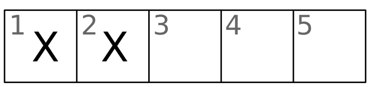
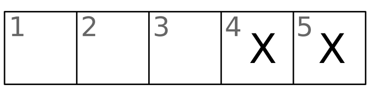
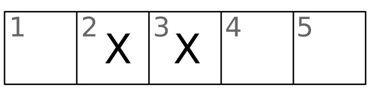
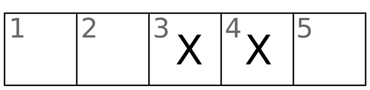
The success of the strategy described above depends on the fact that we were playing as the "first player". There are a few questions that we might ask ourselves:

(1) Is there a different winning strategy for the first player that starts with a different first move?

(2) Is it possible that the second player has a winning strategy as well?

We will explore these questions on the next page.

(1) Let's consider all possible first moves that the first player could make and explore what may happen next in the game.

First Move	**Explanation**
 or 	If we start by placing one game piece on square 1, then our opponent can place two game pieces on squares 3 and 4. We are then forced to choose between square 2 and square 5, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing one game piece on square 5, we can get a similar result.
 or 	If we start by placing one game piece on square 2, then our opponent can place two game pieces on squares 4 and 5. We are then forced to choose between square 1 and square 3, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing one game piece on square 4, we can get a similar result.
	If we start by placing one game piece on square 3, then we already know we are guaranteed a win in this game. *This was the first move in the winning strategy discussed on the previous page.*
 or 	If we start by placing two game pieces on squares 1 and 2, then our opponent can place one game piece on square 4. We are then forced to choose between square 3 and square 5, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing two game pieces on squares 4 and 5, we can get a similar result.
 or 	If we start by placing two game pieces on squares 2 and 3, then our opponent can place one game piece on square 4. We are then forced to choose between square 1 and square 5, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing two game pieces on squares 3 and 4, we can get a similar result.

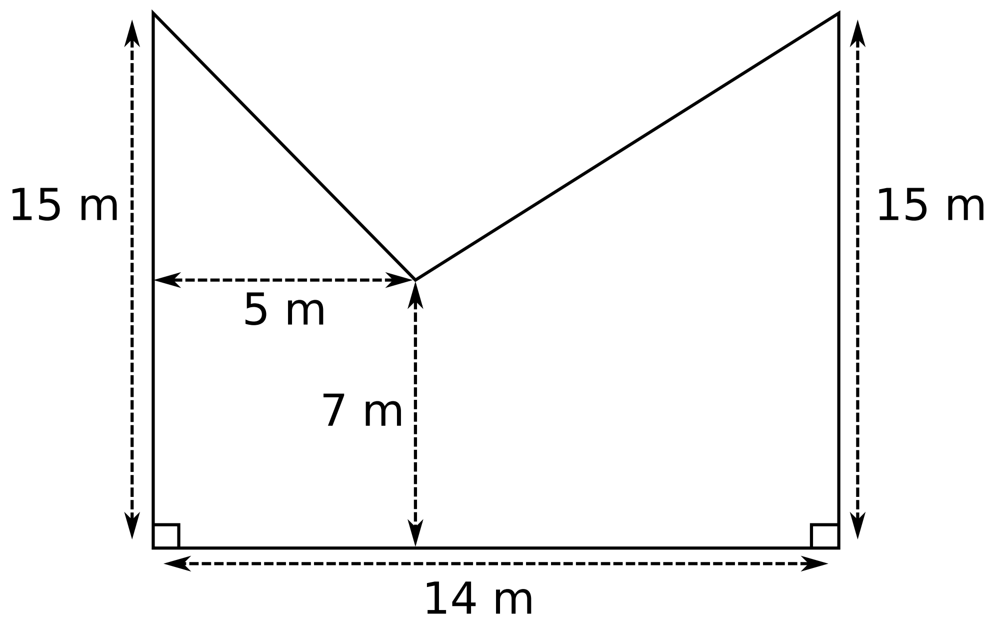
We can argue that every winning strategy for the first player must start with placing one piece on square 3. If the first player starts with any other move, then the second player can win the game as explained in the work above.

(2) The second player cannot also have a winning strategy for this game. It is definitely possible for the second player to win the game (and you may have won games as the second player), but the second player cannot guarantee a win. They have to hope that the first player either does not know the winning strategy described earlier or makes a mistake along the way.

CEMC at Home

Grade 7/8 - Tuesday, April 7, 2020
Floor Plan Areas

Shay asked his two sons, Dexter and Bennett, to find the area of their basement floor. Dexter and Bennett measured their basement and noted the dimensions shown below. What is the area of their basement?



Follow-up Questions:

1. There is more than one way to decompose (break up) the shape above into familiar shapes. Can you find a few different ways to do this?
2. Let's pretend that Dexter and Bennett forgot to measure and label the 5 m dimension in their diagram. Can they still determine the area of their basement without this measurement?
3. Make a floor plan for a room, an apartment, or one floor of a house. Measure any dimensions that you want to use to calculate the area of the floor. Did you actually need all of the dimensions that you measured, or could you have calculated the area with fewer dimensions known?

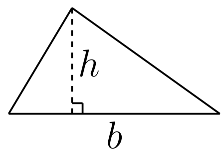
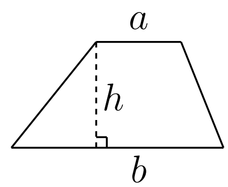
See the next page for a reminder of the area formulas for some familiar shapes, and some extra problems for practice.

More Info:

Check out the CEMC at Home webpage on Wednesday, April 8 for a solution to Floor Plan Areas.

For more practice calculating the area of composite shapes (like the shape above), check out this lesson in the CEMC Courseware.

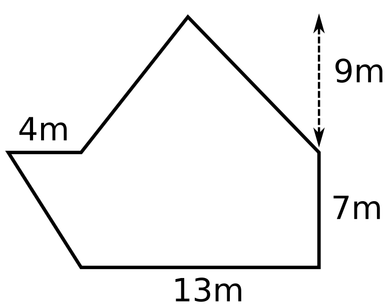
Area Formulas:

Rectangle	Triangle	Trapezoid
		
$A = \ell \times w$	$A = \frac{1}{2}(b \times h)$	$A = \frac{1}{2}(a + b) \times h$

More Practice:

1. Brie-Ann wants to place patio stones on a rectangular pathway towards her house. The pathway has dimensions of 14 m by 4 m. If each patio stone measures 0.5 m by 0.5 m, how many patio stones are needed to cover the pathway?

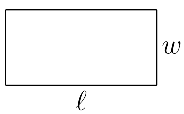
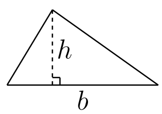
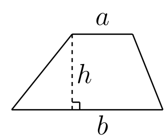
2. Can you calculate the area of the following shape given only the information below? Is there any extra information that you need?



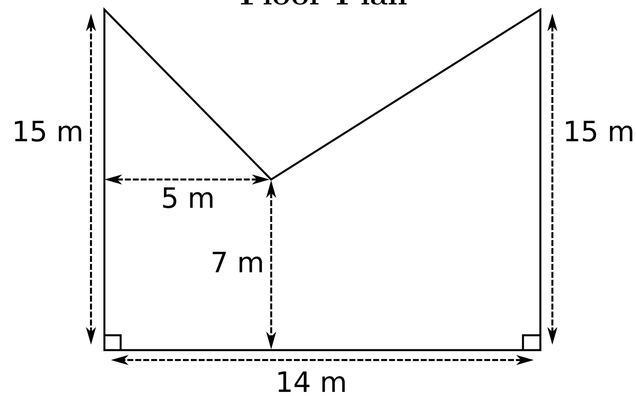
CEMC at Home

Grade 7/8 - Tuesday, April 7, 2020
Floor Plan Areas - Solution

Here is a reminder of the floor plan and the area formulas.

Rectangle	Triangle	Trapezoid
		
$A = \ell \times w$	$A = \frac{1}{2}(b \times h)$	$A = \frac{1}{2}(a + b) \times h$

Floor Plan



Answer: The total area of the floor is 154 m^2.

We will show different ways to calculate this area in the discussion that follows.

Follow-up Question 1: There are many different ways to decompose the shape above into familiar shapes like triangles, rectangles and trapezoids. Two different ways to do this are given here. Did you find these? Did you find others?

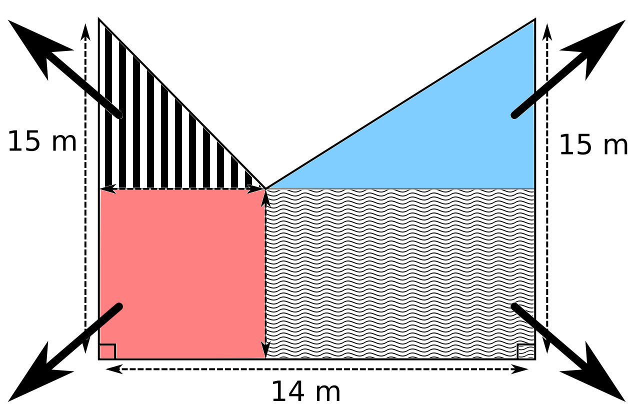
Method 1: Break the shape into two triangles and two rectangles.

Triangle 1

The base is 5 m.
The height is $15 - 7 = 8$ m.

Triangle 2

The base is $14 - 5 = 9$ m.
The height is $15 - 7 = 8$ m.



Rectangle 1

The length is 5 m and the width is 7 m.

Rectangle 2

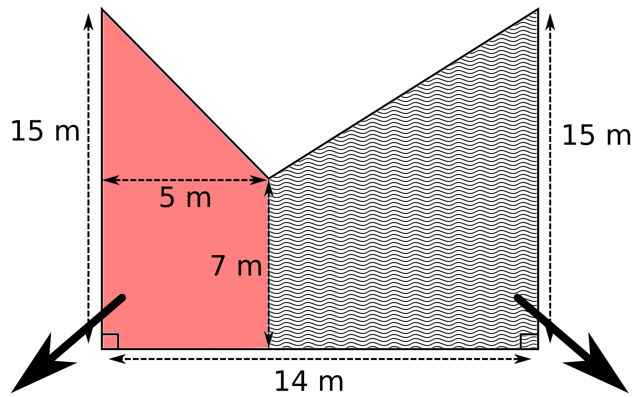
The width is 7 m.
The length is $14 - 5 = 9$ m.

The area of the composite shape is the sum of the areas of the two triangles and the two rectangles.

Shape	Area
Triangle 1	$\frac{1}{2}(b \times h) = \frac{1}{2}(5 \times 8) = 20$
Triangle 2	$\frac{1}{2}(b \times h) = \frac{1}{2}(9 \times 8) = 36$
Rectangle 1	$\ell \times w = 5 \times 7 = 35$
Rectangle 2	$\ell \times w = 9 \times 7 = 63$

Since $20 + 36 + 35 + 63 = 154$, the total area of the floor is 154 m^2.

Method 2: Break the shape into two trapezoids.



Trapezoid 1

The parallel sides are 15 m and 7 m long.
The height is 5 m.

Trapezoid 2

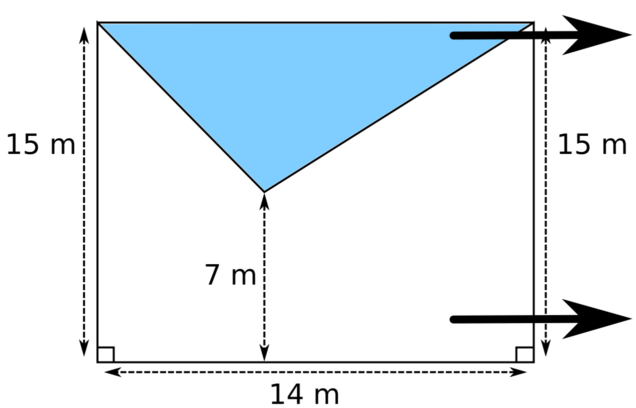
The parallel sides are 15 m and 7 m long.
The height is $14 - 5 = 9$ m.

The area of the composite shape is the sum of the areas of the two trapezoids.

Shape	Area
Trapezoid 1	$\frac{1}{2}(a + b) \times h = \frac{1}{2}(15 + 7) \times 5 = 55$
Trapezoid 2	$\frac{1}{2}(a + b) \times h = \frac{1}{2}(15 + 7) \times 9 = 99$

Since $55 + 99 = 154$, the total area of the floor is 154 m^2.

Follow-up Question 2: It is still possible to determine the area of the basement without knowing the measurement of 5 m. To do this, we think of the shape as a big rectangle with a triangle cut out.



Triangle

The base is 14 m.
The height is $15 - 7 = 8$ m.

Rectangle

The length is 14 m.
The width is 15 m.

The area of the composite shape is the area of the rectangle minus the area of the triangle.

Shape	Area
Triangle	$\frac{1}{2}(b \times h) = \frac{1}{2}(14 \times 8) = 56$
Rectangle	$\ell \times w = 14 \times 15 = 210$

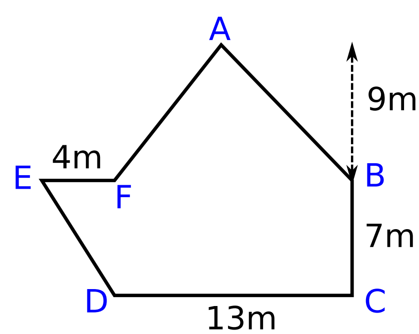
Since $210 - 56 = 154$, the total area of the floor is 154 m^2.

More Practice:

1. Brie-Ann wants to place patio stones on a rectangular pathway towards her house. The pathway has dimensions of 14 m by 4 m. If each patio stone measures 0.5 m by 0.5 m, how many patio stones are needed to cover the pathway?

Answer: Brie-Ann needs 224 patio stones.

2. Can you calculate the area of the following shape given only the information below? Is there any extra information that you need?



Answer: Notice that we added labels to the corners of the shape. We could calculate its area if we knew the following were all true.

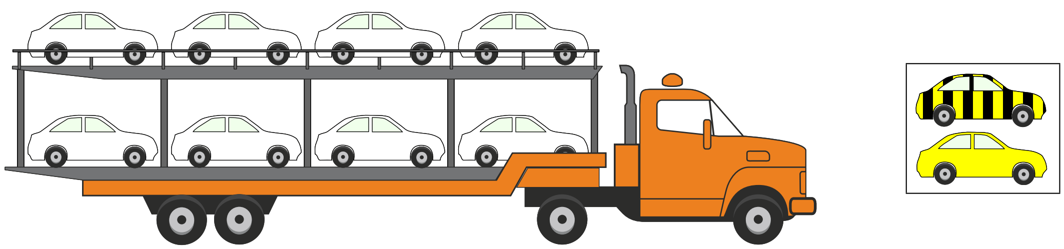
- $\angle BCD$ is a right angle.
- The line connecting E and B passes through F and is parallel to line DC.
- $\angle EFD$ is a right angle.

CEMC at Home

Grade 7/8 - Wednesday, April 8, 2020
Car Transportation

Question 1: A car manufacturer produces cars in solid colours as well as patterns. A new striped car comes from a manufacturing line every 7 minutes. A new solid yellow car comes from another manufacturing line every 4 minutes. Both manufacturing lines start working at the same time. A driver parks the cars on the back of a large transport truck in the order the cars leave their respective manufacturing lines. The top floor of the transport truck is loaded first.

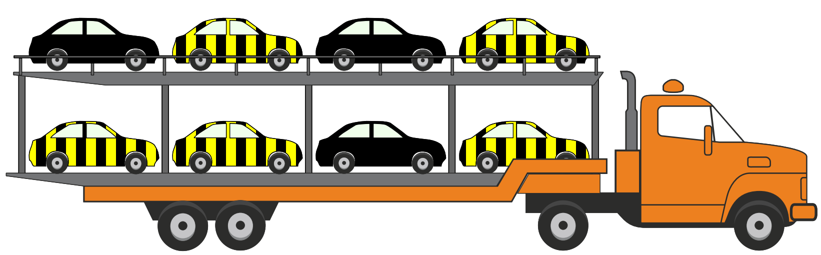
Which of the cars in the loaded transport truck below are striped and which are yellow?



Need help getting started?

Try using objects like coins, paper clips, or pieces of paper to represent the cars leaving their respective manufacturing lines. This is called a simulation. Create a model or a table to summarize what the loading truck looks like at various time intervals.

Question 2: A new solid black car comes from a third manufacturing line every n minutes, where n is an integer between 5 and 10, inclusive. A second transport truck is loaded with only black cars and striped cars using the same method as in Question 1. After the truck is fully loaded it looks like the truck below. What is the value of n?



More Info:

Check out the CEMC at Home webpage on Thursday, April 9 for a solution to Car Transportation.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

Many industries, including car manufacturing, are highly *automated*, and this automation relies on computers to control and coordinate production. As such, the production needs to be carefully planned and synchronized so that various demands (availability of transport trucks, the need for particularly designed items to go on a particular truck, etc.) can be managed. The need to understand, create, manage, and improve automated systems is a real-world example of computer science being applied.

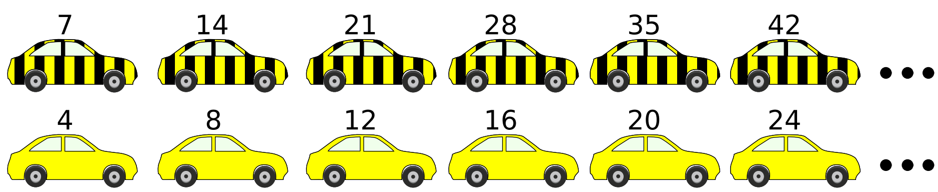
CEMC at Home

Grade 7/8 - Wednesday, April 8, 2020
Car Transportation - Solution

Solution to Question 1

We will start by recording the times at which the first few cars leave their respective manufacturing lines. We will record the elapsed time, starting our clock at the time both lines start working.

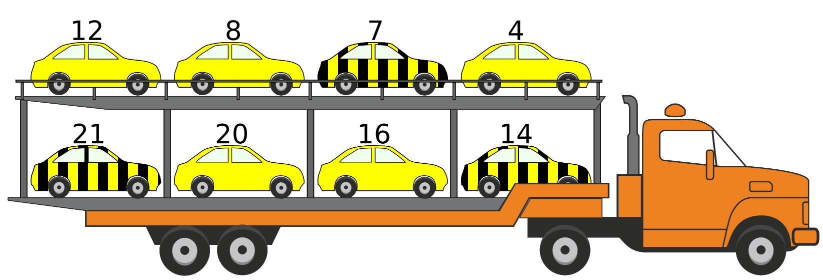
The striped cars come from the manufacturing line every 7 minutes. This means there will be a striped car at 7 minutes, another striped car at 14 minutes, another one at 21 minutes, and so on. The yellow cars come from the manufacturing line every 4 minutes. This means there will be a yellow car at 4 minutes, another yellow car at 8 minutes, another one at 12 minutes, and so on.



The cars are loaded onto the transport truck, filling the top level first and then the bottom level. We can determine the order in which these cars go into the truck by looking at the number (representing the time, in minutes) appearing above each car. From our work above, we can be sure that the first eight times at which a car is released are as follows:

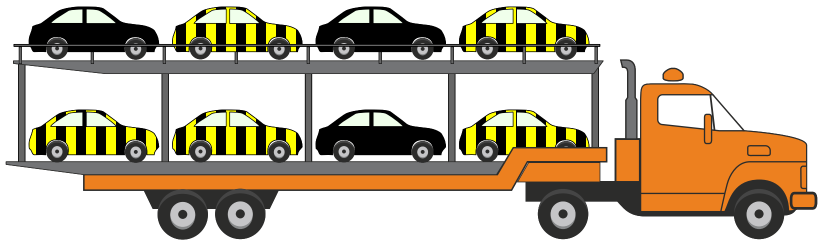
Time	4	7	8	12	14	16	20	21
Colour	Yellow	Striped	Yellow	Yellow	Striped	Yellow	Yellow	Striped

Keeping track of the colour of the car attached to each of the times above, we see that the transport truck will end up looking like this:



Solution to Question 2

Remember that the striped cars come from the manufacturing line every 7 minutes and that the black cars come from the manufacturing line every n minutes. Notice that the first car that was loaded into the truck in Question 2 is a striped car.

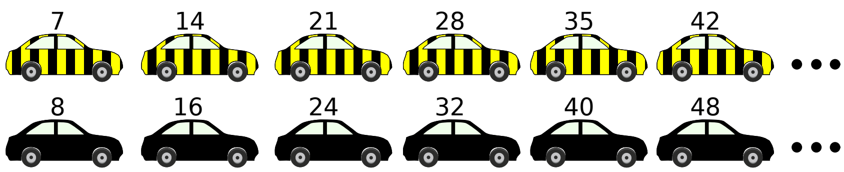


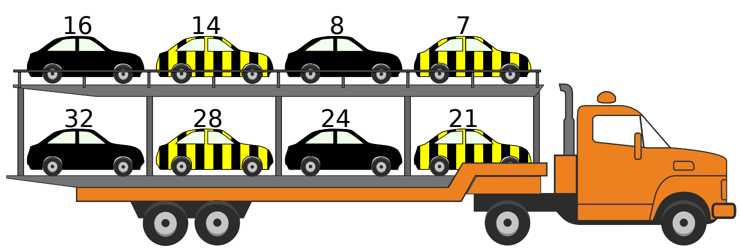
This means that n cannot be less than 7, as otherwise the first car would have to be black. Since n is an integer from the list $5, 6, 7, 8, 9, 10$, we know that n must be equal to 7, 8, 9, or 10.

The case where $n = 7$ is special, and we will discuss this at the end.

Now that we have narrowed our answer down to only a few possibilities, we may choose to proceed by checking what would happen in each of the different cases and see which values of n lead to the correct picture. We do similar work as in the solution to Question 1 for each possible value of n.

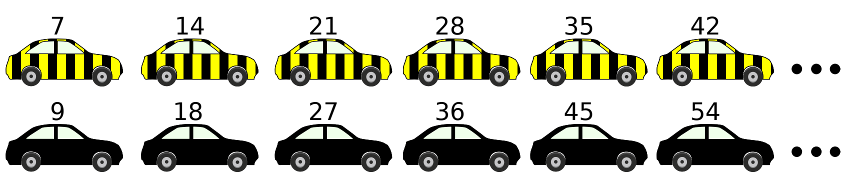
Option 1: $n = 8$

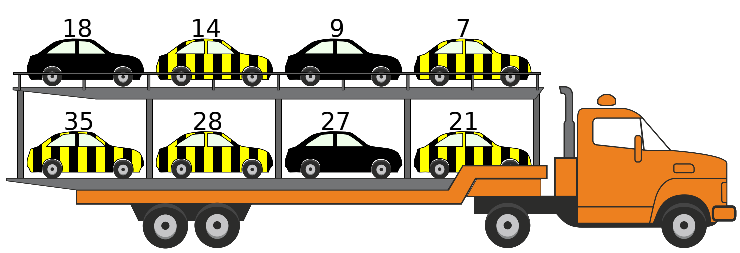




This is not the picture we are looking for, so n cannot be equal to 8.

Option 2: $n = 9$

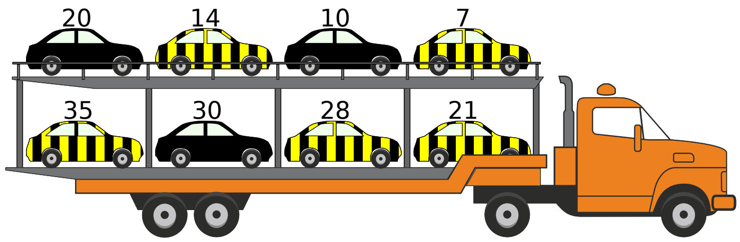




This shows us that $n = 9$ does produce the correct picture.

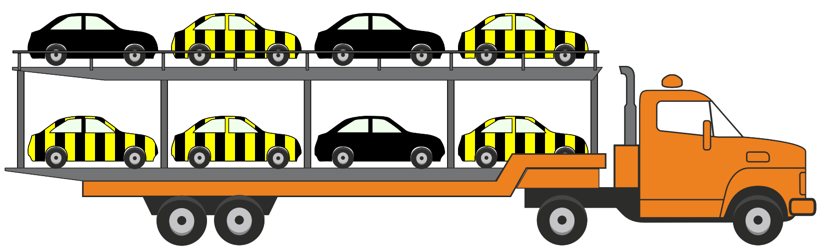
Option 3: $n = 10$

We can verify that $n = 10$ also results in the wrong final picture, so n cannot be equal to 10.



Option 4: $n = 7$

Notice that, in this case, a car comes from both manufacturing lines every 7 minutes. We are not told which line will get priority when two cars arrive at the same time, but no matter what rule you make, you cannot end up with the correct final picture (which is shown again below).



If $n = 7$, then the first eight cars must be split evenly with four striped and four black, which is not the case in the picture for Question 2 above. Therefore, n cannot be equal to 7.

From our work, we conclude that the value of n must be 9, and so the black cars come from the third manufacturing line every 9 minutes.

Follow-up Question: Can you explain why $n = 9$ is the only positive integer n that will result in the given transport truck picture? We have explained why 9 is the only number from the list $5, 6, 7, 8, 9, 10$ that gives the correct picture, but can you explain why no integer outside of this list can work either?

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 9, 2020
What's Not to Love?

At the start of the school year, students in Mr. Pi's class were asked the following question: "Do you love Math?" They were only allowed to answer "yes" or "no", and everyone had to answer. Of the 30 students in the class, 21 answered "yes" and 9 answered "no".

That day, with every student present, the probability of randomly selecting a student who answered the question "yes" was $\frac{21}{30} = \frac{7}{10}$ and the probability of randomly selecting a student who answered the question "no" was $\frac{9}{30} = \frac{3}{10}$.

However, on one particular morning later in the year, the following information was known about the class:

- at least one of the students who had answered "yes" was absent and at least one of the students who had answered "no" was absent;
- more than half of the class was present; and
- the probability of randomly selecting a student who had answered the question "yes" was $\frac{3}{4}$.

Is there enough information to determine how many students were absent that particular morning? If yes, how many students were absent? If no, explain why not.



More Info:

Check the CEMC at Home webpage on Thursday, April 16 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 16.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
What's Not to Love?

Problem

At the start of the school year, students in Mr. Pi's class were asked the following question: "Do you love Math?" They were only allowed to answer "yes" or "no", and everyone had to answer. Of the 30 students in the class, 21 answered "yes" and 9 answered "no". So, with every student present, the probability of randomly selecting a student who answered the question "yes" was $\frac{21}{30} = \frac{7}{10}$ and the probability of randomly selecting a student who answered the question "no" was $\frac{9}{30} = \frac{3}{10}$.

However, on one particular morning later in the year, the following information was known about the class: at least one of the students who had answered "yes" was absent and at least one of the students who had answered "no" was absent; more than half of the class was present; and the probability of randomly selecting a student who had answered the question "yes" was $\frac{3}{4}$.

Is there enough information to determine how many students were absent that particular morning? If yes, how many students were absent? If no, explain why not.

Solution

Since at least one student from each of the two groups was absent, there were at least 2 students absent and at most 28 students present. Also, the maximum number of students who said "yes" would be $21 - 1 = 20$ and the maximum number of students who said "no" would be $9 - 1 = 8$. More than half the class was present so at least 16 students were present.

Since the probability of randomly selecting a student who answered "yes" was $\frac{3}{4}$, then the probability of randomly selecting a student who answered "no" was $\frac{1}{4}$.

We are looking for any number from 16 to 28 which is divisible by 4, so that when we find $\frac{3}{4}$ and $\frac{1}{4}$ of this number our result is a whole number. The numbers from 16 to 28 that are divisible by 4 are 16, 20, 24 and 28.

The following chart shows the results which are possible using the given information. There are 3 valid solutions that satisfy the given information. Therefore, there is not enough given information to determine the number of students who were absent that particular morning. The last solution in the chart in not valid. If the number present was 28, then 21 of those present would have answered the question "yes". But at least one student who answered "yes" was absent so the maximum number of students who answered "yes" would have been 20.

Number Present	Number Absent	Number who said "yes"	Number who said "no"	Valid / Not Valid
16	$30 - 16 = 14$	$\frac{3}{4} \times 16 = 12$	$\frac{1}{4} \times 16 = 4$	Valid
20	$30 - 20 = 10$	$\frac{3}{4} \times 20 = 15$	$\frac{1}{4} \times 20 = 5$	Valid
24	$30 - 24 = 6$	$\frac{3}{4} \times 24 = 18$	$\frac{1}{4} \times 24 = 6$	Valid
28	$30 - 28 = 2$	$\frac{3}{4} \times 28 = 21$	$\frac{1}{4} \times 28 = 7$	Not Valid

To Think About: Is there another piece of information that Mr. Pi could have provided so that two of the three valid answers could be eliminated leaving only one valid answer?