Grade 6 Math Circles  
Wednesday, March 3, 2021  
*Exponents - Problem Set*

1. Evaluate.

(a) \(5^3\)  
(b) \(\left( \frac{101}{4} \right)^0\)  
(c) \(2^6\)  
(d) \(1^{27}\)  
(e) \(4^4\)  
(f) \(0^{56}\)  
(g) \(13^2\)  
(h) \(\phi^1\)

2. Write the following as powers. *(Hint: For part (c), try factoring the number)*

(a) \(333 \times 333 \times 333\)  
(b) 91 to exponent 5  
(c) 216  
(d) 5 to the fourth power  
(e) 100 squared  
(f) \(7 \times 11 \times 11 \times 7 \times 11 \times 7 \times 7\)

3. Use the given base to write the following numbers as powers.

(a) 4096, base = 4  
(b) 625, base = 5  
(c) 1000, base = 10  
(d) 512, base = 2  
(e) 289, base = 17  
(f) 81, base = 3

4. Express 1 as a power that has a base not equal to 1.

5. Evaluate.

(a) \((-30)^4\)  
(b) \(47^{-1}\)  
(c) \(2^{-6}\)  
(d) \((-1)^{38}\)  
(e) \((15)^{-3}\)  
(f) \(\left( -\frac{7}{6} \right)^5\)  
(g) \((-88)^0\)  
(h) \(\left( -\frac{4}{25} \right)^{-2}\)
6. Fill in the table summarizing different properties of exponents.

<table>
<thead>
<tr>
<th>Property</th>
<th>Explanation</th>
<th>Example</th>
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<tbody>
<tr>
<td>Base of 1</td>
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<tr>
<td>Base of 0</td>
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<td>Exponent of 1</td>
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<td>Exponent of 0</td>
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<td>Negative Exponent</td>
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<td>Negative Base</td>
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7. Consider the fraction $\frac{16}{81}$.

(a) Write $\frac{16}{81}$ as a power with a negative base.

(b) Write the fraction as a power with a negative base and exponent.

8. Use exponent rules to simplify the following expressions. Note that part (g)-(h) are challenge questions.
9. Fill in the blanks with <, >, or = to complete the inequality.

(a) $2^4 \text{______} 4^2$
(b) $6^{-3} \text{______} 6^3$
(c) $99^0 \text{______} 0^{45}$
(d) $(-1)^{27} \text{______} 1^{27}$
(e) $7^{-3} \text{______} 7^{-1}$
(f) $4^{-6} \text{______} (-4)^{-7}$

10. The human brain has \( \left( \frac{10^{15}}{(10^{23})^{2^2}} \right) \) neuronal connections. Express this number as a single power.

11. According to the Big Bang Model, the universe has \( \left( \frac{(5^{26})(5^{9})}{5^{34}} \right)^{1450} \times (10^{11})^2 \) stars. Simplify the expression.

12. Tom had three hamsters at the beginning of the year. After 4 weeks, he now has 243 hamsters. Assuming weekly exponential growth, how much is the population exponentially increasing by every week?

13. A bacteria population quadruples in size every 10 minutes. If the initial bacteria population is 7 specimens, what is the population after 1 hour?

14. Rio created a computer algorithm which can process an exponential amount of data and reduce processing time. For example, let \( x \) represent the initial amount of data the algorithm can process in one minute. Then, it can process \( x^2 \) amount of data in 2 minutes. Rio calculates that the algorithm is able to process 400 GB of data in 2 minutes. How long would it take to process 3,200,000 GB of data?

15. Magic Morning Coffee shares that 15% of the caffeine in their drink is consumed by the human body every hour. If a teacher had 1 cup of coffee, or 128 mg of caffeine, at 10 AM this morning, how much caffeine is left in their system at 5 PM?

16. The **half-life** of a substance is the amount of time it takes for the quantity of the substance to decrease to half its original value. For example, gallium-67 is a substance used in nuclear medicine with a half life of 80 hours. If a scientist has 288 grams of gallium-67 at the beginning of an experiment, and 72 grams at the end, how long was the experiment?