Intermediate Math Circles
Triangles, Circles and Area
Problem Set

Express your answers as simplified exact numbers. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

**Exercise 1**
Three circles are centred at $C, O$ and $D$ as shown. $AB$ is a diameter of the larger circle and $C$ and $D$ are on $AB$. $OA$ and $OB$ are diameters of the smaller circles. The larger circle has a diameter of 12. Find the area of the shaded region.

**Exercise 2**
The circle with centre $O$ has a radius of 2. Points $A$ and $B$ are on the circle and $\angle AOB = 90^\circ$ as shown. Find the area of the shaded region.

**Exercise 3**
Square $ABCD$ is inscribed in the circle with centre $O$ and radius 5 as shown. Find the area of the shaded region.

A *square is inscribed in a circle if all four vertices of the square lie on the circle.*

**Exercise 4**
A circle is centred at $O$. $OABC$ is a rhombus with $A, B,$ and $C$ on the circle. If $\angle AOC = 120^\circ$ and $OA = 10$, find the area of the shaded region. (See if you can solve this problem two ways. One using equilateral triangles and the other using properties of a rhombus.)

**Exercise 5**
Two circles are centred at $C$ and $O$ as shown. The circles intersect at points $A$ and $B$ with $\angle AOB = 60^\circ$ and $\angle ACB = 90^\circ$. The circle centred at $O$ has a radius of $\sqrt{2}$. Find the area of the shaded region.