



## Grade 6 Math Circles

November 28/29 *Solutions*  
*Math Jeopardy*

### Introduction

Questions will vary in difficulty with \$100 questions tending to be the easiest, and \$500 questions tending to be the hardest. Do your best, good luck and have fun!

### Sequence of Probable Events

\$100 What is the probability of rolling a prime number on a 6-sided die?

On a die you can roll any of the 6 possible outcomes 1-6. 3 of those possible numbers are prime: {2,3,5}. Thus there is a probability of  $\frac{3}{6} = \frac{1}{2}$  to roll a prime number on a die.

\$200 You have 4 shirts, 5 pairs of pants and 3 pairs of shoes to choose from. How many outfits can you make? We have to make 3 choices. Choice A: a shirt, Choice B: pair of pants and Choice C: shoes. We have to make Choice A, Choice B and Choice C. Since we make a choice **AND** another choice, we can use product rule. We multiply the number of options for each choice to find all possible combinations. Total combinations =  $4 \times 5 \times 3 = 60$

\$300 What is the next term in the following sequence? {0,1,1,2,3,5,8,13,21,34} This is the famous fibonacci sequence where each term is created by the sum of the two terms before it. For example  $a_3 = a_2 + a_1$  or  $1 + 1 = 2$ . Thus the next term in the sequence will be the sum of 21 and 34, 55.

\$400 What type of sequence is this? What is the 11th term?

$$\{3,9,15,21\}$$

This is an arithmetic sequence as each term has a common difference of six. We can use the formula  $a_n = a_1 + d(n - 1)$  where  $a_1$  is 3 and the common difference  $d$  is 6. From here we find that  $a_{11} = a_1 + d(11 - 10)$

$$a_{11} = 3 + 6(10) = 63$$

The 11th term of the sequence of 63.

\$500 You are hired by your friend to help build a shed for 5 weeks. You are paid \$11 the first week. Every week after you are paid twice as much as the week before. How much money will you make in total?

The pay for each week: \$11, \$22, \$44, \$88, \$176. Adding up the pay from each week, you will make \$341 in total.

## The Powers of Algebra

\$100 Addition is repeated addition. Exponentiation is repeated \_\_\_\_\_?

Exponentiation is repeated multiplication.

\$200 Solve for the  $x$  in the following equation:  $15 + 2x = 7x$

$$15 = 7x - 2x$$

$$15 = 5x$$

$$3 = x$$

\$300 Express the following as a power of 2:

$$\frac{2^4 \times 7^0 \times 2^6}{2^3}$$

Exponent Laws: When multiplying like bases, **add** the exponents. When dividing like bases, **subtract** exponents. Anything to the exponent 0 is 1.

$$2^4 \times 7^0 \times 2^6 = 2^4 \times 1 \times 2^6$$
$$2^{4+6} = 2^{10}$$

Thus

$$\frac{2^4 \times 7^0 \times 2^6}{2^3} = \frac{2^{10}}{2^3}$$

$$\frac{2^{10}}{2^3} = 2^{10-3} = 2^7$$

\$400 Solve for  $x$  in the following equation:

$$x + 3 \times 4 + x = (40 - 6) \div 2$$

Use BEDMAS!!

$$x + 3 \times 4 + x = (40 - 6) \div 2$$

$$x + 12 + x = (34) \div 2$$

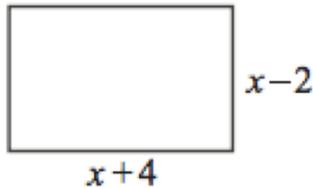
$$12 + 2x = 17$$

$$2x = 17 - 12$$

$$2x = 5$$

$$x = \frac{5}{2}$$

\$500 In the diagram, the perimeter of the rectangle is 56 units. What is its area?



All the sides of the square added up (perimeter) are equal to 56 units.

$$(x + 4) + (x - 2) + (x + 4) + (x - 2) = 56$$

Collecting like terms:

$$4x + 4 + 4 - 2 - 2 = 56$$

$$4x + 4 = 56$$

Algebra Equation, we can solve for x!

$$4x = 52$$

$$x = 13$$

Can use  $x$  to find the sides of our rectangle:

Horizontal Sides:  $x - 2 = 13 - 2 = 11$  units

Vertical Sides:  $x + 4 = 13 + 4 = 17$  units

Area Rectangle:  $17 \times 11 = 187$  units squared

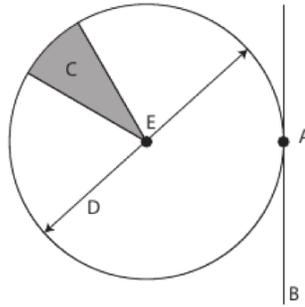
## Running in Circles

\$100 What is the result of dividing the circumference of a circle by its radius?

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

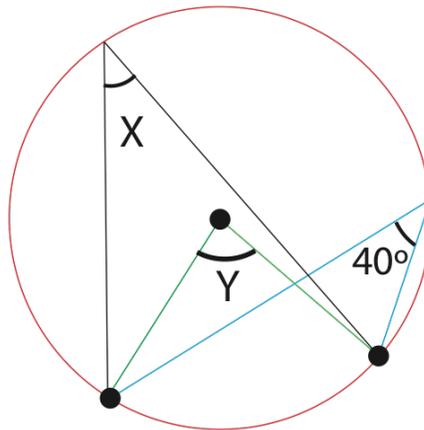
This comes directly from the equation for Circumference of a circle  $C = \pi d$ .

\$200 Correctly Identify each part of the circle:



A: Point, B: Tangent Line, C: Sector, D: Diameter, E: Centre

\$300 Using the diagram find angles  $X$  and  $Y$ :

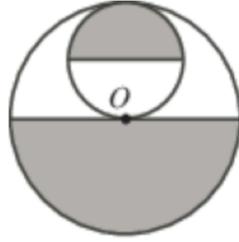


Angles  $40^\circ$ ,  $X$  and  $Y$  are all along the same chord (endpoints). *Angles Subtended by the Same Arc Theorem* tells us that the angles  $X$  and  $40^\circ$  are equal as inscribed angles along the same arc. *Central Angle Theorem* tells us that the central angle  $Y$  is twice as large as corresponding inscribed angles on the same chord  $X$  and  $40^\circ$ .  
 $X = 40^\circ, Y = 80^\circ$

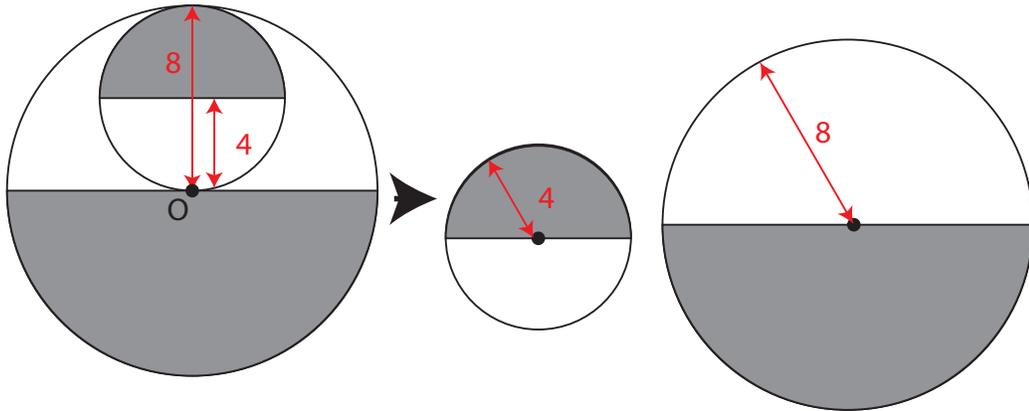
\$400 The circumference of a circle is  $10\pi$ . What is its area?

We know that  $C = \pi d$  thus we can find the diameter of the circle. If  $C = \pi d = 10\pi$  then  $d$  must be 10. If diameter is 10 then the radius would be half that, 5. The area of a circle of radius 5:  $A = \pi 5^2 = 25\pi$

\$500 In the diagram, each circle is divided into two equal areas and  $O$  is the centre of the larger circle. The radius of the larger circle is 8. What is the total area of the shaded regions?



The large circle has radius 8, the small circle has half of the radius of the large circle. Radius of small circle is 4.



Area Shaded is half the area of each circle.

$$Area_{Large} = \frac{1}{2} \times \pi 8^2 = 32\pi$$

$$Area_{Small} = \frac{1}{2} \times \pi 4^2 = 8\pi$$

$$\text{Total Area Shaded} = 32\pi + 8\pi = 40\pi$$

## A Number of Secrets

\$100 What sets of numbers do the following numbers belong to?

$$7, \sqrt{2}$$

7: Natural, Whole, Integer, Rational, Real

$\sqrt{2}$ : Irrational, Real

\$200 How many prime numbers are there between 10 and 30? **6 prime numbers: (11,13,17,19,23,29)**

\$300 Decode this message that has been encrypted with an Atbash Cipher.

Gsv ovzevh ziv uzoormt

The leaves are falling

\$400 Decrypt the following message with Pigpen Cipher:

└┐┌┐> >┐┌┐┐┐┐

A	B	C	J	K	L
D	E	F	M	N	O
G	H	I	P	Q	R

S	W
T	X
U	Y
V	Z

ALEX TREBEK

\$500 Police are attempting to identify the name of a doctor. They have found a document with what they believe to be the doctors name on it but it has been encrypted with a Caesar Shift. The encryption of the doctors name looks as follows:

Ym. Wdgg Rdggdvhn

What is the shift number of the Caesar Cipher? What is the name of the Doctor?

Using the fact that this is a doctor, we can assume that the name starts with the title Dr. Thus from there we can determine that a shift number of 5 was used (5 letters between r and w for example). Thus we can decode the whole name as Dr. Bill Williams.

## Logically Puzzling

\$100 Two fathers took their sons fishing. Each person caught one fish, but when they returned to camp, there were only 3 fish. How could this be?

A boy, his father, and his grandfather went fishing.

\$200 It is noon and John wants to go get lunch at Subway. But, there is a terrible snowstorm outside and it is impossible for him to drive there. However, the forecast says that the snow will turn into hail and it will hail the rest of the day. How can you determine whether the sun will be shining 36 hours from now?

It will be midnight in 36 hours therefore the sun will not be shining.

\$300 How many people would we need in this room to guarantee that two people have the same birthday?

There are 366 possible birthdays including leap years (someone could be born February 29th). Therefore 366 people could have different birthdays. Only when there are 367 people can we be certain that two people share a birthday.

\$400 Given the equivalences, what's the missing number?

$$12 = 6$$

$$6 = 3$$

$$5 = \_$$

Trick Question: The answer is not 2.5. The answer corresponds to the number of letters in each number. Twelve has 6 letters, six has three. Five has 4. The missing number is 4.

\$500 Use exactly four 4s (and no other numbers) to make the numbers 7 and 3. You can use any combination of operations/tools.

For example:  $\times \div + - \sqrt{\quad}$

$$3 : (4 + 4 + 4) \div 4$$

$$7 : 4 + 4 - \frac{4}{4}$$

## Gauss Contest

\$100 Which of the following numbers lies between 3 and 4 on a number line?

(A)  $\frac{5}{2}$  (B)  $\frac{11}{4}$  (C)  $\frac{11}{5}$  (D)  $\frac{13}{4}$  (E)  $\frac{13}{5}$

(Source: 2018 Gauss (Grade 7), #6)

Coverting each of the improper fractions to a mixed fraction, we get  $\frac{5}{2} = 2\frac{1}{2}$ ,  $\frac{11}{4} = 2\frac{3}{4}$ ,  $\frac{11}{5} = 2\frac{1}{5}$ ,  $\frac{13}{4} = 3\frac{1}{4}$ ,  $\frac{13}{5} = 2\frac{3}{5}$ .

Of the five answers given, the number that lies between 3 and 4 on a number line is  $3\frac{1}{4}$  or  $\frac{13}{4}$ .

\$200 Chris and Pat are playing catch. Standing 1 m apart, Pat first throws the ball to Chris and then Chris throws the ball back to Pat. Next, standing 2 m apart, Pat throws to Chris and Chris throws back to Pat. After each pair of throws, Chris moves 1 m farther away from Pat. They stop playing when one of them misses the ball. If the game ends when the 29th throw is missed, how far apart are they standing and who

misses catching the ball? (Source: 2005 Gauss (Grade 7), #19 )

At each distance, two throws are made: the 1st and 2nd throws are made at 1 m, the 3rd and 4th are made at 2 m, and so on, with the 27th and 28th throws being made at 14 m.

Therefore, the 29th throw is the first throw made at 15 m.

At each distance, the first throw is made by Pat to Chris, so Chris misses catching the 29th throw at a distance of 15 m.

\$300 Which of the following is closest to one million seconds?

(A) 1 day (B) 10 days (C) 100 days (D) 1 year (E) 10 years

(Source: 2006 Gauss (Grade 7), #19)

In one minute, there are 60 seconds.

In one hour, there are 60 minutes, so there are  $60 \times 60 = 3600$  seconds.

In one day, there are 24 hours, so there are  $24 \times 3600 = 86,400$  seconds.

In 10 days there are therefore 864,000 seconds which is the closest option to 1,000,000 seconds

\$400 An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 4, 6, 8 and 1, 4, 7, 10 are arithmetic sequences.

In the grid shown, the numbers in each row must form an arithmetic sequence and the numbers in each column must form an arithmetic sequence. The value of  $x$  is

(A) 37 (B) 28 (C) 36 (D) 43.75 (E) 46

1			
4			25
7			$x$
10		36	

(Source: 2013 Gauss (Grade 7), #22)

We use labels,  $m$  and  $n$ , in the fourth row of the grid, as shown. Then, 10,  $m$ , 36,  $n$  are four terms of an arithmetic sequence. Since 10 and 36 are two terms apart in this sequence, and their difference is  $36 - 10 = 26$ , the constant added to one term to obtain the next term in the fourth row is  $\frac{26}{2}$  or 13. That is,  $m = 10 + 13 = 23$ , and  $n = 36 + 13 = 49$ . (We confirm that the terms 10, 23, 36, 49 do form an arithmetic sequence.)

1			
4			25
7			$x$
10	$m$	36	$n$

In the fourth column, 25 and  $n$  (which equals 49) are two terms apart in this sequence, and their difference is  $49 - 25 = 24$ . Thus, the constant added to one term to obtain the next term in the fourth column is  $\frac{24}{2}$  or 12. That is,  $x = 25 + 12 = 37$  (or  $x = 49 - 12 = 37$ ). The completed grid is as shown.

1	5	9	13
4	11	18	25
7	17	27	37
10	23	36	49

\$500 The operation  $\Delta$  is defined so that  $a\Delta b = a \times b + a + b$ . For example,  $2 \Delta 5 = 2 \times 5 + 2 + 5 = 17$ . If  $p \Delta 3 = 39$ , the value of  $p$  is what?

(A) 13 (B) 12 (C) 9 (D) 10.5 (E) 18

(Source: 2015 Gauss (Grade 8), #18)

$$p \Delta 3 = p \times 3 + p + 3 = 39$$

$$4p + 3 = 39$$

$$4p = 36$$

$$p = 9$$

## Final Jeopardy

\$1500 Lara ate  $\frac{1}{4}$  of a pie and Ryan ate  $\frac{3}{10}$  of the same pie. The next day Cassie ate  $\frac{2}{3}$  of the pie that was left. What fraction of the original pie was not eaten?

Together, Lara and Ryan ate  $\frac{1}{4} + \frac{3}{10} = \frac{5}{20} + \frac{6}{20} = \frac{11}{20}$  of the pie.

Therefore,  $1 - \frac{11}{20} = \frac{9}{20}$  of the pie remained.

The next day, Cassie ate  $\frac{2}{3}$  of the pie that remained.

This implies that  $1\frac{2}{3} = \frac{1}{3}$  of the pie that was remaining was left after Cassie finished eating.

Thus,  $\frac{1}{3}$  of  $\frac{9}{20}$ , or  $\frac{3}{20}$  of the original pie was eaten.