Intermediate Math Circles
Fall 2019
Fun With Inequalities

Puneet Sharma

The Department of Applied Mathematics
Faculty of Mathematics
University of Waterloo

November 6, 2019
What Happened Last Week?

- We looked at sets, interval notation, bracket notation, and representing interval on real number line.
What Happened Last Week?

- We looked at sets, interval notation, bracket notation, and representing interval on the real number line.
- We looked at the definition of "less than or equal to".
What Happened Last Week?

- We looked at sets, interval notation, bracket notation, and representing interval on real number line
- We looked at the definition of “less than or equal to”
- We looked at some properties of “less than or equal to”
What Happened Last Week?

- We looked at sets, interval notation, bracket notation, and representing interval on real number line.
- We looked at the definition of “less than or equal to”
- We looked at some properties of “less than or equal to”
- We proved one of them
What Happened Last Week?

- We used those properties to help us solve linear inequalities with one variable (Algebraically and representation on number line)

Plan for week 2

- Solve Absolute Value Inequalities (Single Variable)
- Solve Rational Inequalities (Single Variable)
- Prove some properties of the Absolute Value Function.
What Happened Last Week?

- We used those properties to help us solve linear inequalities with one variable (Algebraically and representation on number line)
- We solved word problems using linear inequalities with one variable

Plan for week 2

- Solve *Absolute Value Inequalities (Single Variable)* and *Rational Inequalities (Single Variable)*.
- Prove some properties of the Absolute Value Function.
What is *absolute value*?

**Definition**

The *absolute value* $|b|$ of a real number $b$ is defined to be $b$ if $b$ is positive or zero, and to be $-b$ if $b$ is negative.
What is *absolute value*?

**Definition**

The *absolute value* $|b|$ of a real number $b$ is defined to be $b$ if $b$ is positive or zero, and to be $-b$ if $b$ is negative.

What does this look like in “math speak”? 

$$|b| = \begin{cases} 
    b & \text{if } b \geq 0 \\
    -b & \text{if } b < 0 
\end{cases}$$
Solving Absolute Value Inequalities (Single Variable)

What is absolute value?

**Definition**

The absolute value $|b|$ of a real number $b$ is defined to be $b$ if $b$ is positive or zero, and to be $-b$ if $b$ is negative.

What does this look like in “math speak”?

$$|b| = \begin{cases} 
    b & \text{if } b \geq 0 \\
    -b & \text{if } b < 0
\end{cases}$$

Another cool way of expressing absolute value is as follows

$$|b| = \sqrt{b^2}$$
Another way to think about *absolute value* is the distance from a *special point*. Sometimes that *special point* is zero, sometimes it is non-zero, and sometimes there are multiple *special point*. To find these *special point* we need to set what’s contain in each absolute value to zero and solve.
Solving Absolute Value Inequalities (Single Variable)

Examples

Solve each of the following equations and inequalities.

1. \(|x| = 12\)
2. \(|x| \geq 5\)
3. \(|x + 6| = 5\)
4. \(|x - 4| \geq 1\)
5. \(|x - 3| + |x + 6| < 13\)
Proving some properties of the Absolute Value Function

Challenge

Prove the following:

1. \(|-x| = x|
2. \(|x| - |y| \leq |x - y|
3. \(||x| - |y|| \leq |x - y|)

Puneet Sharma
Math Circles
November 6, 2019 7 / 12
Before we deal with inequalities let’s work with equalities first.

Practice

Solve each of the following equations.

1. \( \frac{4}{x} = \frac{5}{6} \)
2. \( \frac{\text{-}2}{x + 4} = \frac{3}{x - 1} \)

The most important thing to remember when solving rational equalities is:

**DON’T DIVIDE BY ZERO!**
Solving rational inequalities is what you would expect. It is similar to solving rational equalities, except for that pesky property 7.

Just like when solving rational equalities you need to be aware when the denominator can be zero and exclude those values from your answer.

**Example**

Solve the inequality \( \frac{4}{x} < -\frac{5}{6} \) algebraically.
Where is the error?

Example

For our example, can you spot the error in the following solution?

\[
\frac{4}{x} < -\frac{5}{6}; \quad x \neq 0
\]

\[
4(6) < -5(x)
\]

\[
24 < -5x
\]

\[
\frac{24}{-5} > \frac{-5x}{-5}
\]

\[
-\frac{24}{5} > x
\]

\[
x < -\frac{24}{5}
\]

\[
x < -4.8
\]
Challenge

Solve the inequality \( \frac{-2}{x + 4} \geq \frac{3}{x - 1} \) algebraically.
Thank you!