Intermediate Math Circles
Wednesday March 20, 2019
Introduction to Vectors I

A vector is used to describe such things as velocity and force.

A scalar only has ______________.
A vector has both ______________ and ______________.

Example of scalars ______________, ______________, and ______________.
Example of vectors ______________, ______________, and ______________.

We will name the vector in two ways.

1. ______________- label starting from the tail to the tip.

2. ______________ - label with a single lower case level.

We define ______________ vectors as vectors that have the same magnitude and direction. Conversely if two vectors have the same magnitude and direction then they are ______________. So two directed line segments with the same length and the same direction represent the same vector.

Which of the following pairs of vectors appear to be equal?

a)  

b)  

c)  

Note the two vectors in part b) are said to be ______________ vectors because they have the same length but they are in the opposite directions.

Given square \(ABCD\) labelled as shown. State a) two pairs of equal vectors and b) two pairs of opposite vectors.

a)  

b)  

1
SUM OF VECTORS
John walks from $A$ to $B$. He then walks from $B$ to $C$.

The definition of the sum of vectors is:
Suppose $\vec{a}$ and $\vec{b}$ are any two vectors. Choose points $O$ and $A$ so that $\vec{a} = \overrightarrow{OA}$. Choose a point $B$ so that $\vec{b} = \overrightarrow{AB}$. The sum $\vec{a} + \vec{b}$, of $\vec{a}$ and $\vec{b}$ is represented by $\overrightarrow{OB}$.

In the following diagram, $ABCD$ is a parallelogram. Express $\overrightarrow{CA}$ as the sum of two vectors in as many ways as possible.

SCALAR MULTIPLICATION
$k > 0$
In general, if we multiply $\vec{a}$ by a scalar $k$, $k > 0$, then $k\vec{a}$ is a vector in the same direction of $\vec{a}$ but $k$ times as long. This is written as $k\vec{a} = \vec{a}$. 

$k < 0$
In general, if we multiply $\vec{a}$ by a scalar $k$, $k < 0$, then $k\vec{a}$ is a vector in the opposite direction of $\vec{a}$ but $|k|$ (absolute value of $k$) times as long. This is written as $k\vec{a} = -\vec{a}$.

Given $\vec{a}$, draw
a) $3\vec{a}$.  b) $-2\vec{a}$
VECTOR SUBTRACTION

To subtract a vector, add its _____________.

VECTOR COMBINATION

Given \( \vec{a} \) and \( \vec{b} \) draw the following:

a) \( \vec{a} + \vec{b} \)  
b) \( \vec{a} - \vec{b} \)  
c) \( 2\vec{a} - 3\vec{b} \)

Example 1:

Federico walks 50m north (from \( A \) to \( B \)). He then walks 50m east (from \( B \) to \( C \)). What is the resultant displacement? (i.e. \( \overrightarrow{AC} \)) This means we need to find the magnitude of \( \overrightarrow{AC} \) (We write this as \(|\overrightarrow{AC}|\)). Therefore \(|\overrightarrow{AB}| = 50\) and \(|\overrightarrow{BC}| = 50\). We will also need to find the direction of \( \overrightarrow{AC} \).
Example 2:
Federico walks 30m north (from A to B). He then walks 40m east (from B to C). What is the resultant displacement? (i.e. \( \vec{AC} \))

![Diagram showing Federico's movement](image)

VECTOR PROOFS

1.) Given \( C \) is the midpoint of \( AB \) explain why \( \vec{AC} = \vec{CB} \)

![Diagram showing midpoint](image)

2.) Given \( C \) divides \( AB \) in the ratio 3:1 explain why \( \vec{AC} = \frac{3}{4} \vec{AB} \)

![Diagram showing ratio division](image)

Explanation:
Since \( C \) is on the line segment \( AB \) then \( \vec{AC} \) is in the same direction as \( \vec{AB} \). Therefore \( |\vec{AC}| = 3|\vec{CB}| \) or \( \frac{1}{3}|\vec{AC}| = |\vec{CB}| \).

Now:
\[
|\vec{AB}| = |\vec{AC}| + |\vec{CB}|
\]
\[
|\vec{AB}| = |\vec{AC}| + \frac{1}{3}|\vec{AC}|
\]
\[
|\vec{AB}| = \frac{4}{3}|\vec{AC}|
\]
or
\[
\frac{3}{4}|\vec{AB}| = |\vec{AC}|
\]
therefore \( \vec{AC} = \frac{3}{4} \vec{AB} \)
3.) Given $C$ divides $AB$ in the ratio 3:1 and $O$ is not on $AB$ then express $\overrightarrow{OC}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$

![Diagram]

Solution:
From the previous question we know $\overrightarrow{AC} = \frac{3}{4} \overrightarrow{AB}$ (1)
Here is what else we know
$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ (2)
$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
or $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ (3)

Using (1) and (2) we get, $\overrightarrow{OC} = \overrightarrow{OA} + \frac{3}{4} \overrightarrow{AB}$
Now substituting (3) we get

$$
\overrightarrow{OC} = \overrightarrow{OA} + \frac{3}{4} (\overrightarrow{OB} - \overrightarrow{OA})
= \overrightarrow{OA} + \frac{3}{4} \overrightarrow{OB} - \frac{3}{4} \overrightarrow{OA}
= \frac{1}{4} \overrightarrow{OA} + \frac{3}{4} \overrightarrow{OB}
$$