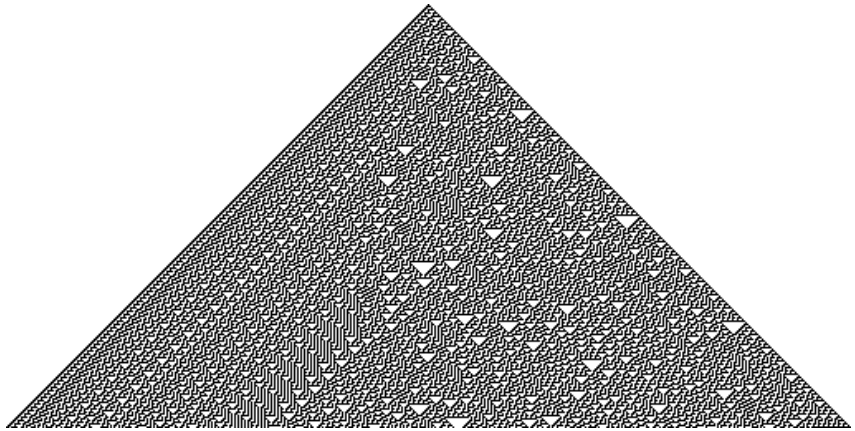
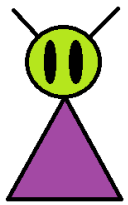
1-Dimensional Cellular Automata



Math Circles 2018
University of Waterloo

Planet Friendship

This is Zoink. He is a Budling from Planet Friendship.



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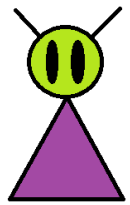

At first, Zoink was alone. But Budlings have the special power of **friendification**!

Planet Friendship

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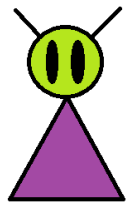

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Planet Friendship

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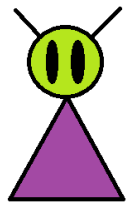
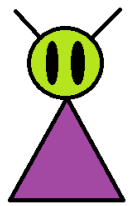


At first, Zoink was alone. But Budlings have the special power of **friendification**!

- Every Budling can create Budlings beside him.
- But if a Budling gets surrounded, he dies!

Day 1



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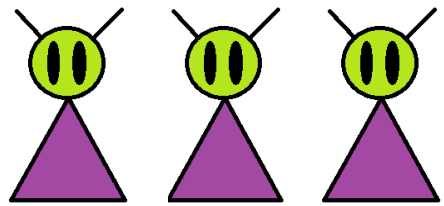

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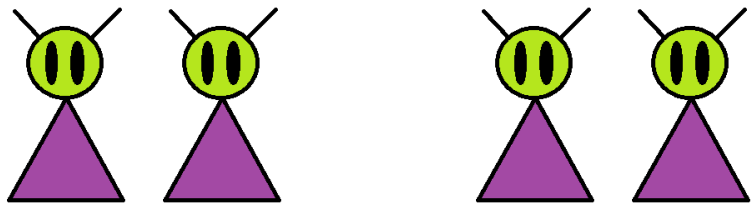

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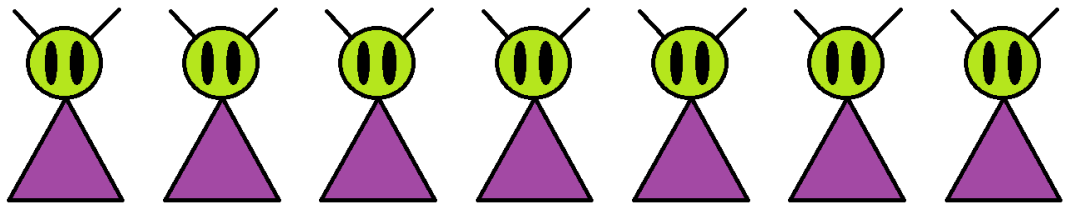

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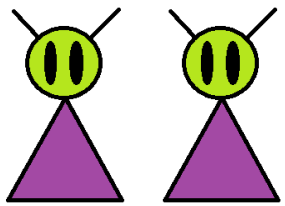

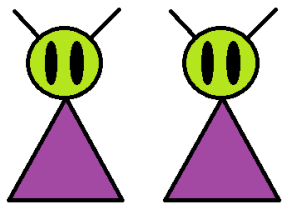

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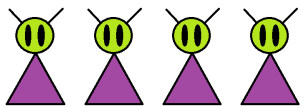
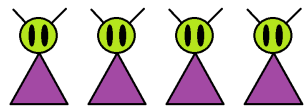
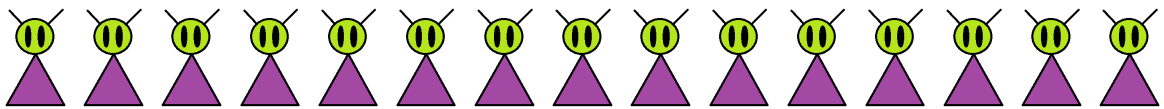




Day 7

Day 8



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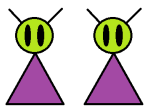

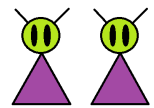

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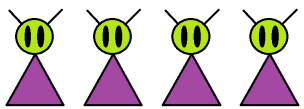
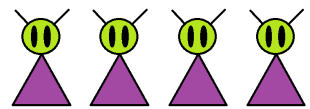
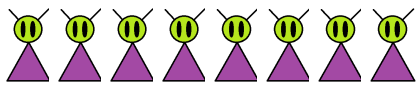
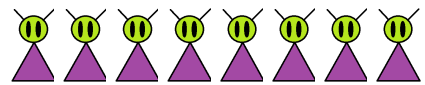




Day 11

Day 12



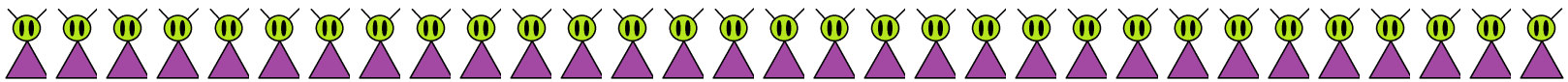


Day 13

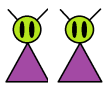
Day 14

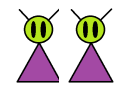
Day 15

Day 16



Day 17





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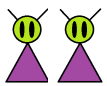

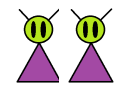

... and so continues the vicious cycle of death on Planet Friendship.

First 16 days on Planet Friendship

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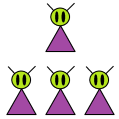

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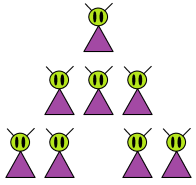

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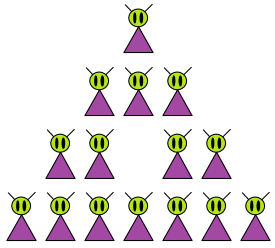

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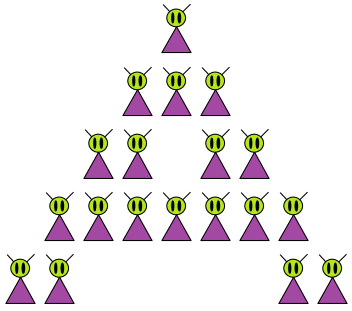

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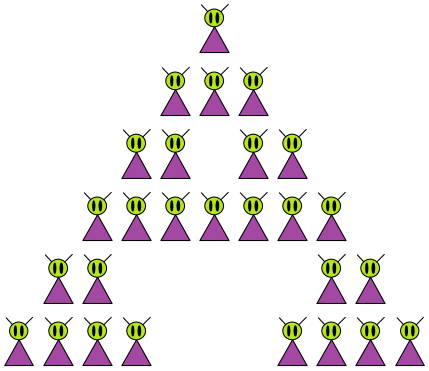

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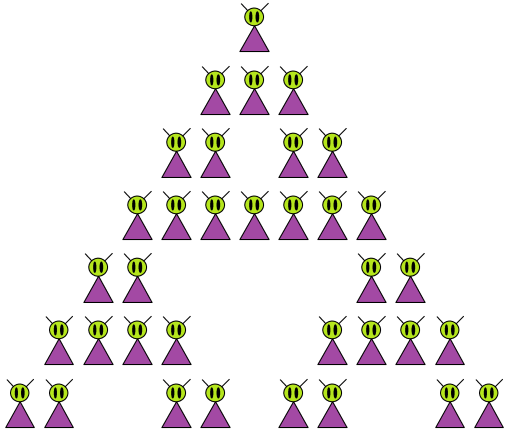

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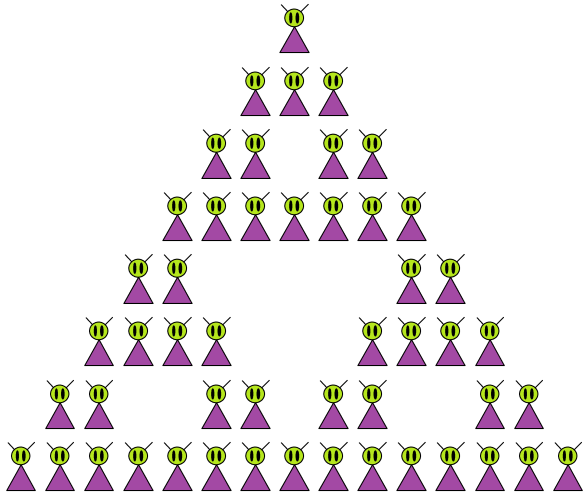

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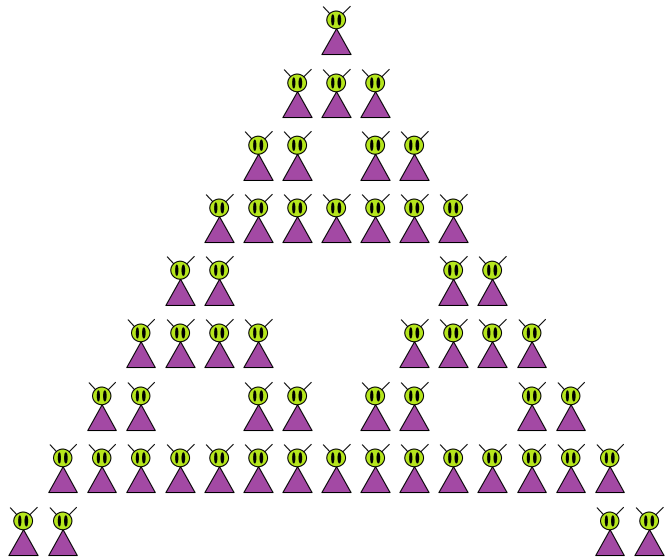

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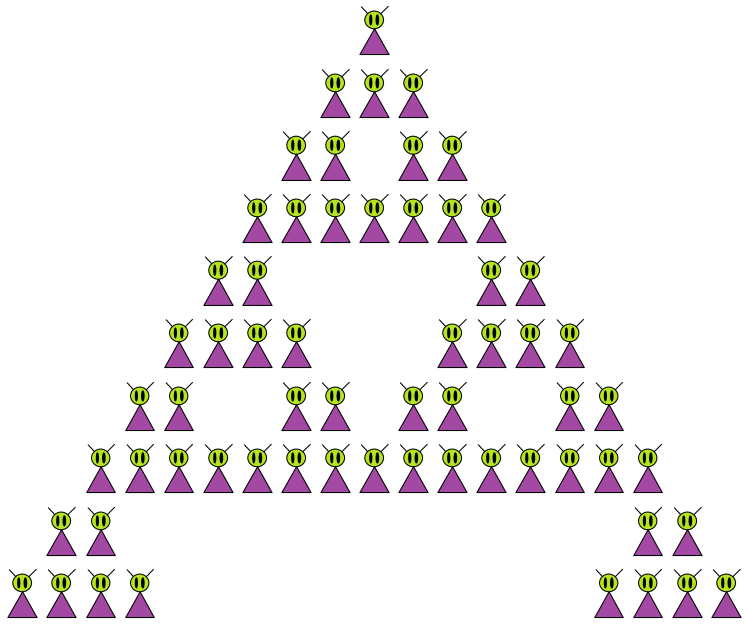

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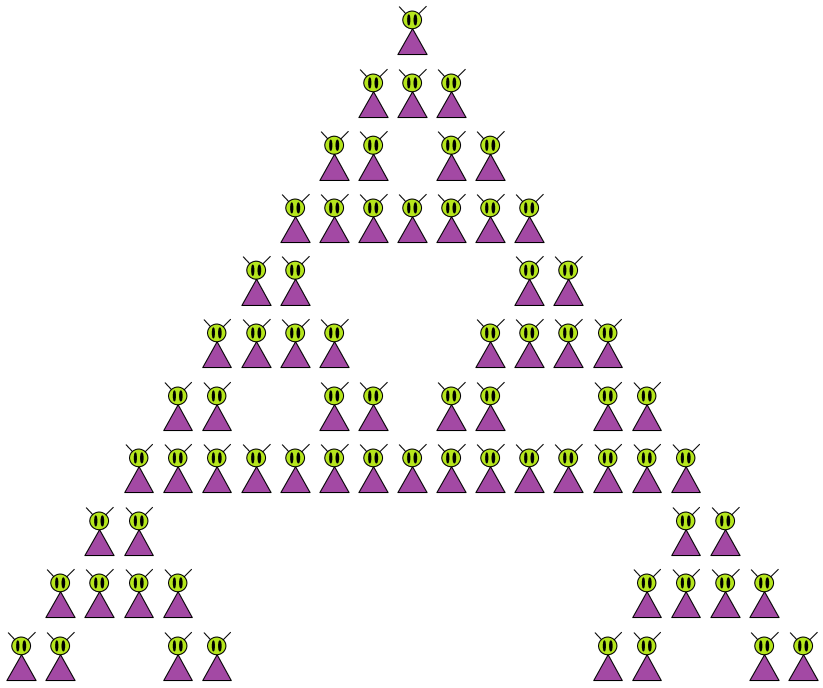

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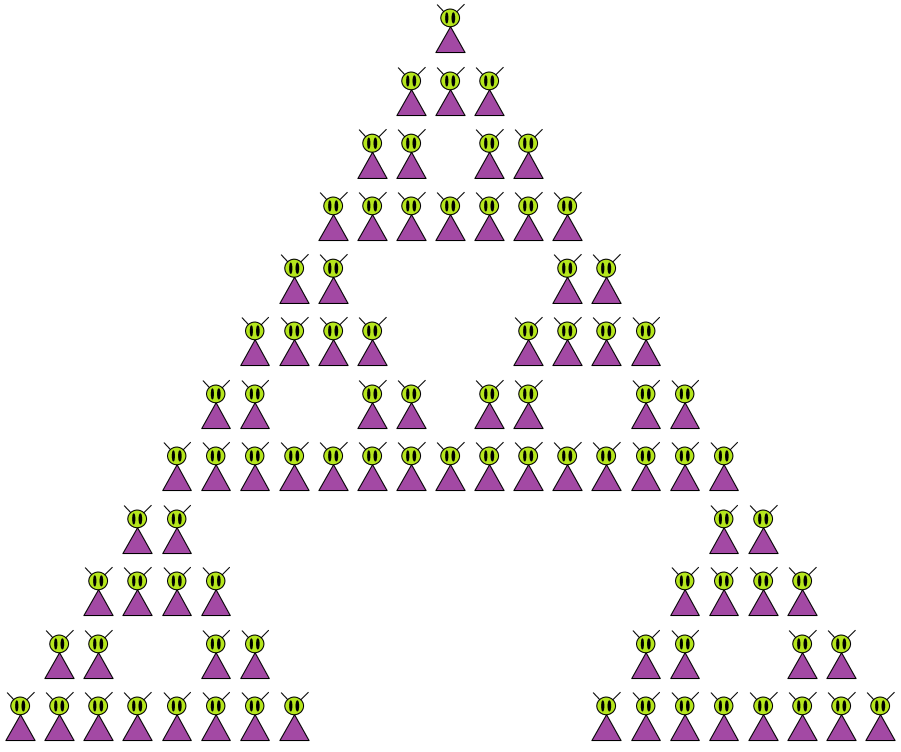

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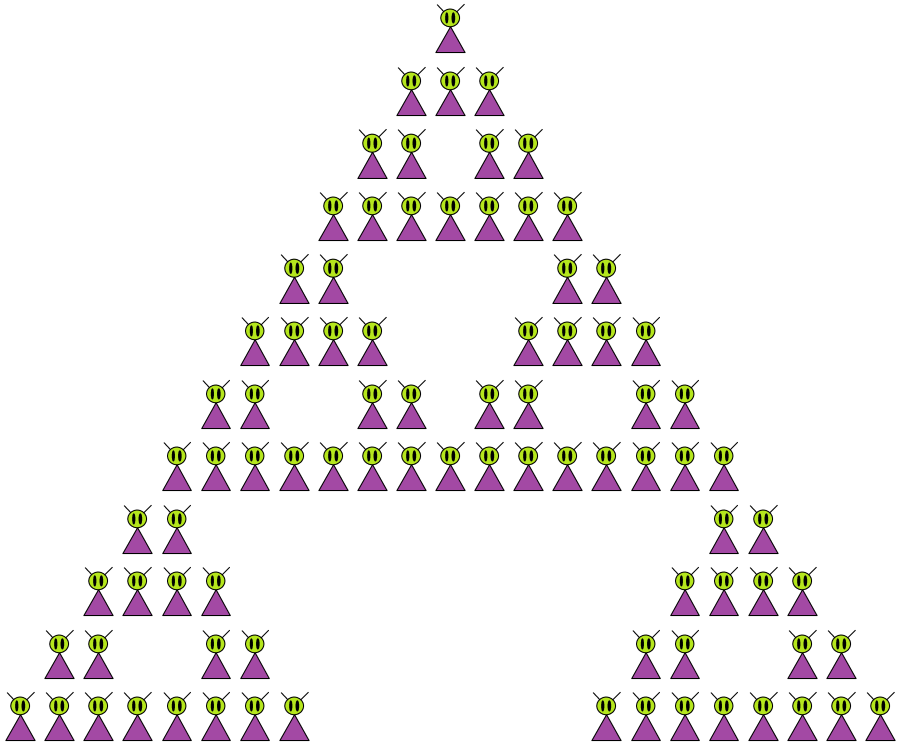

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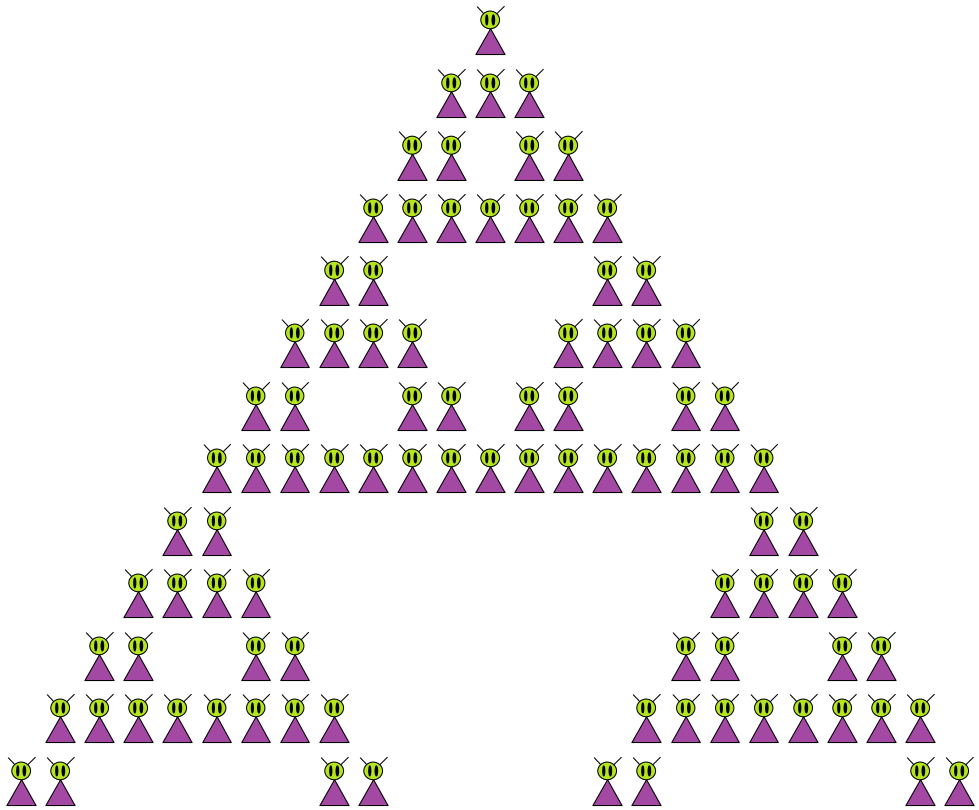

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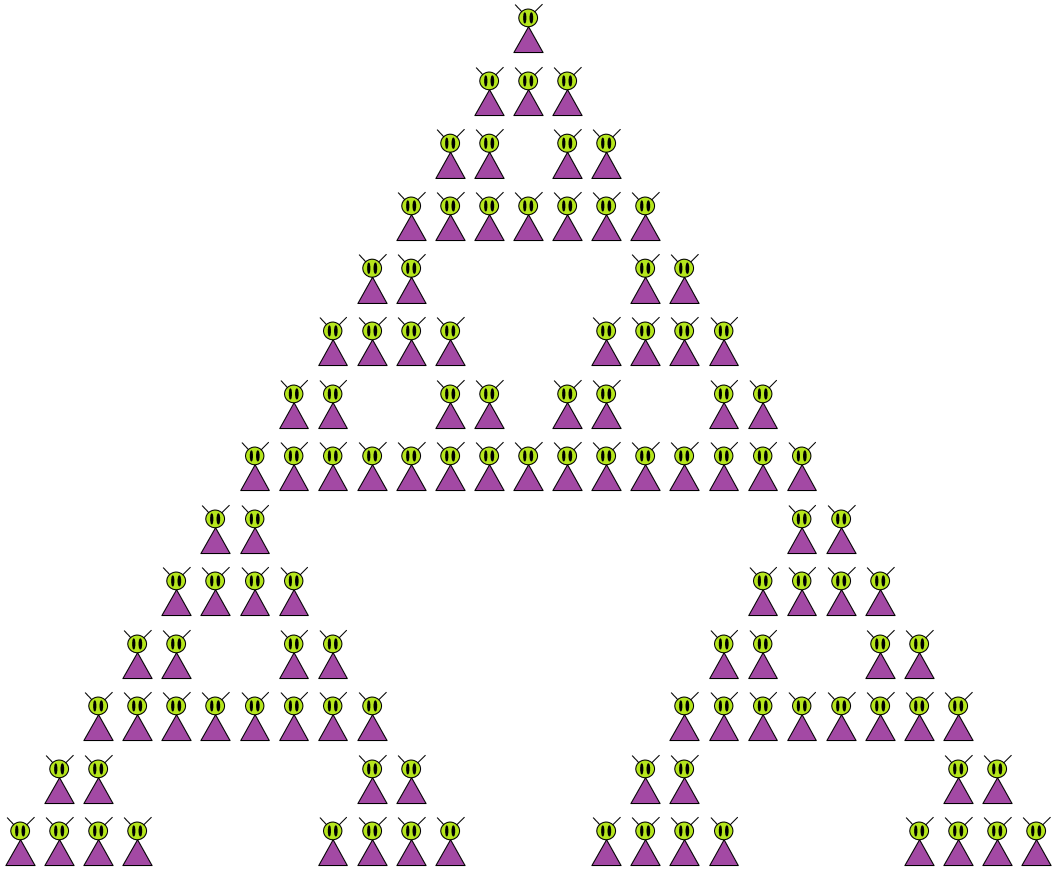

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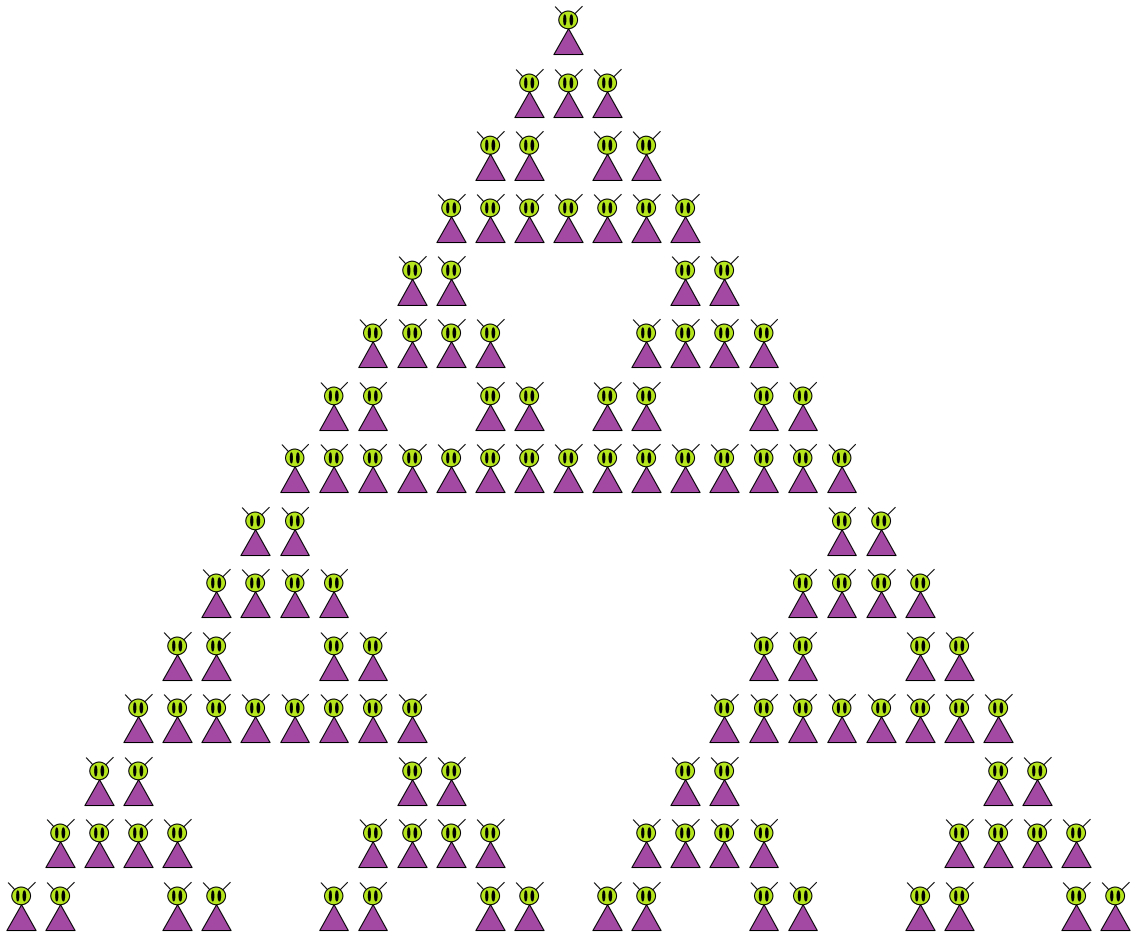

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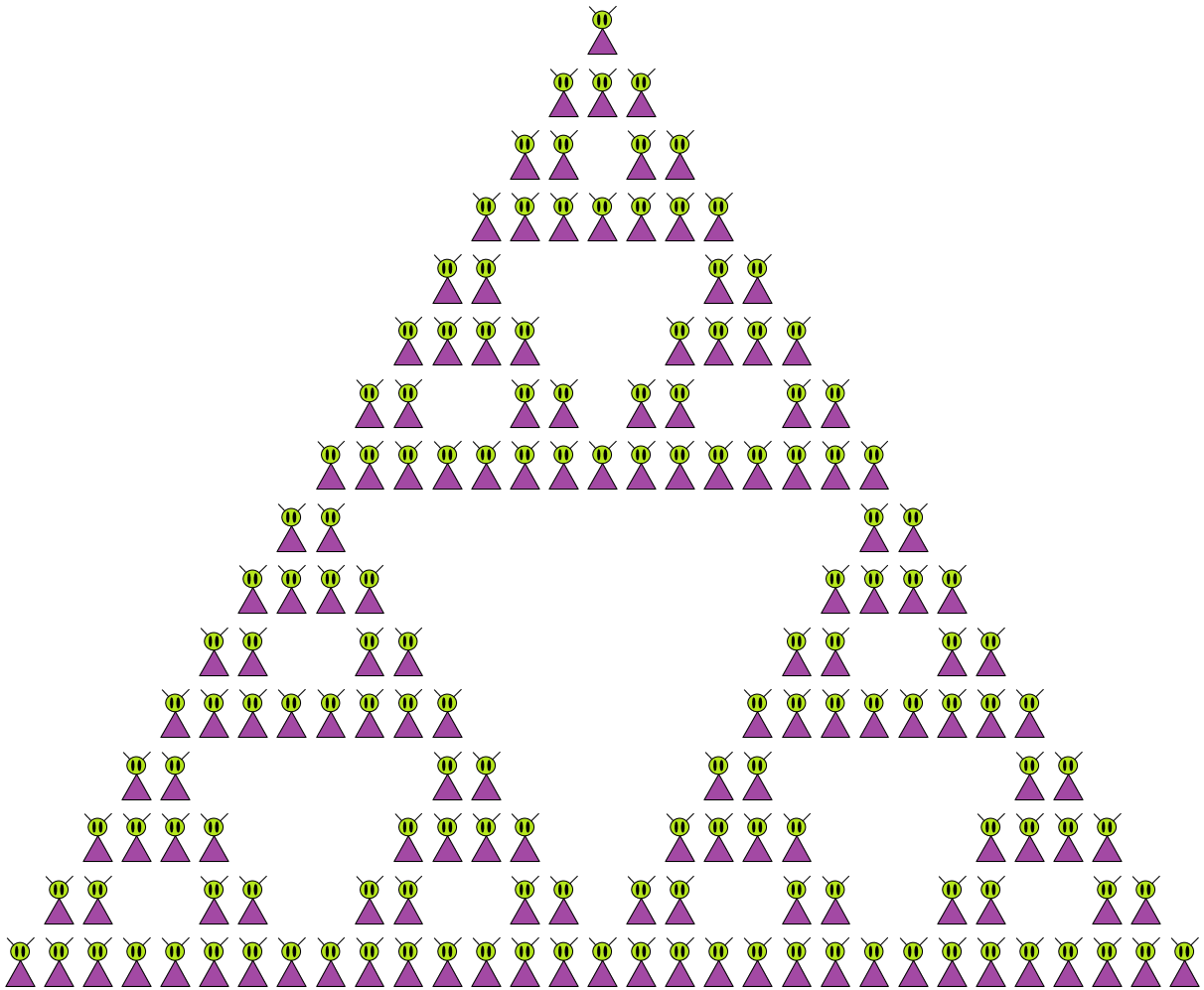

Birth, survival, and death

The rules on Planet Friendship are very precise.

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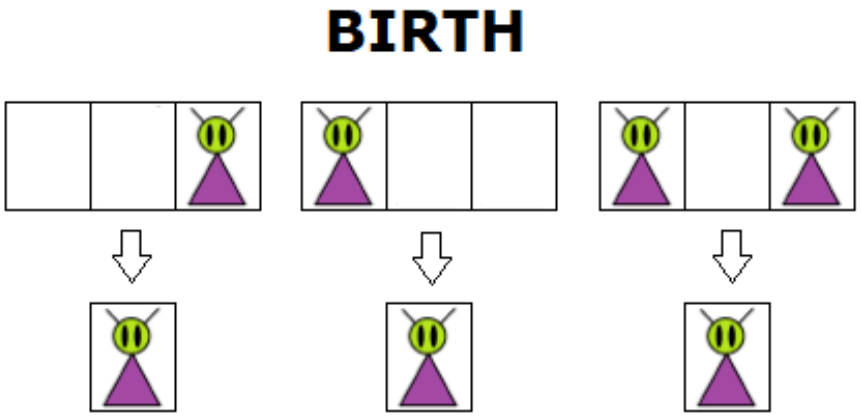

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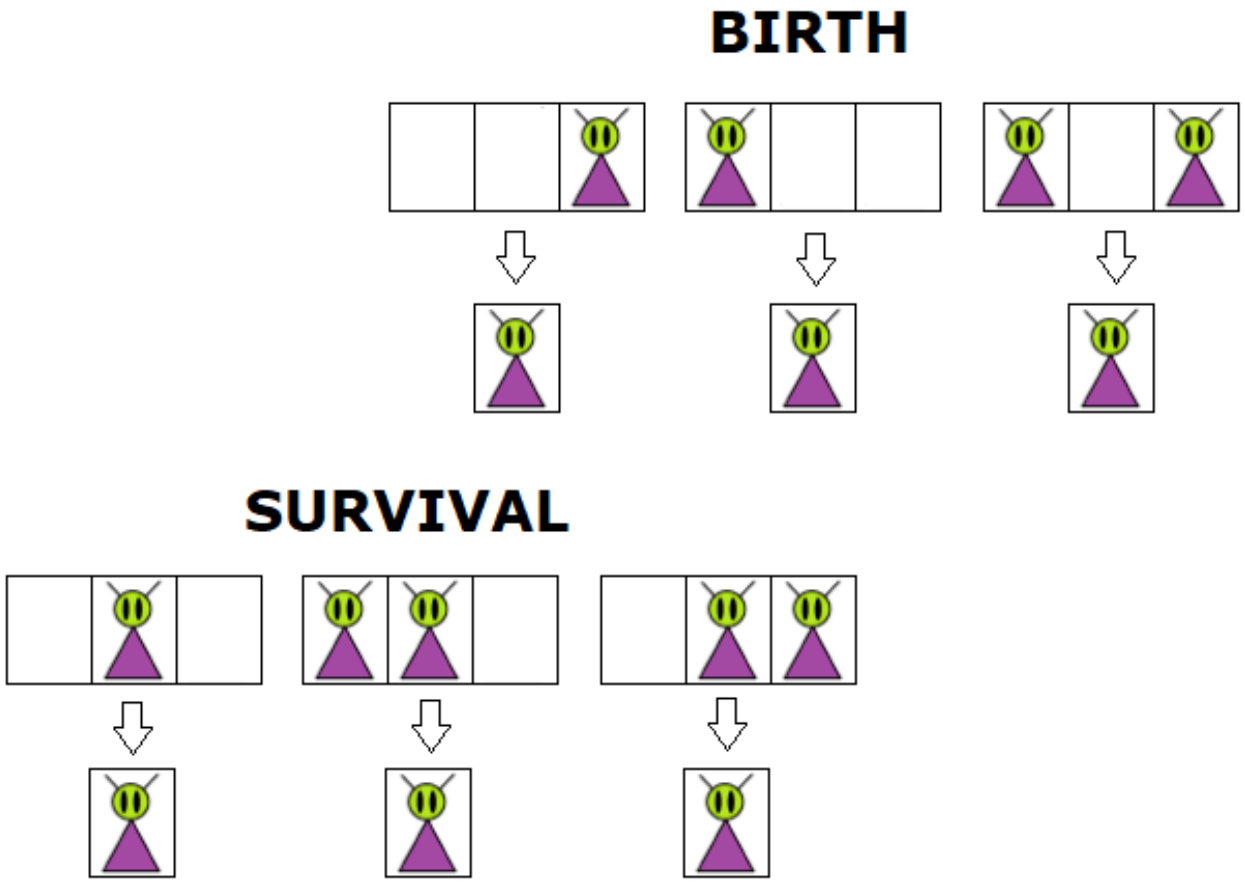

Birth, survival, and death

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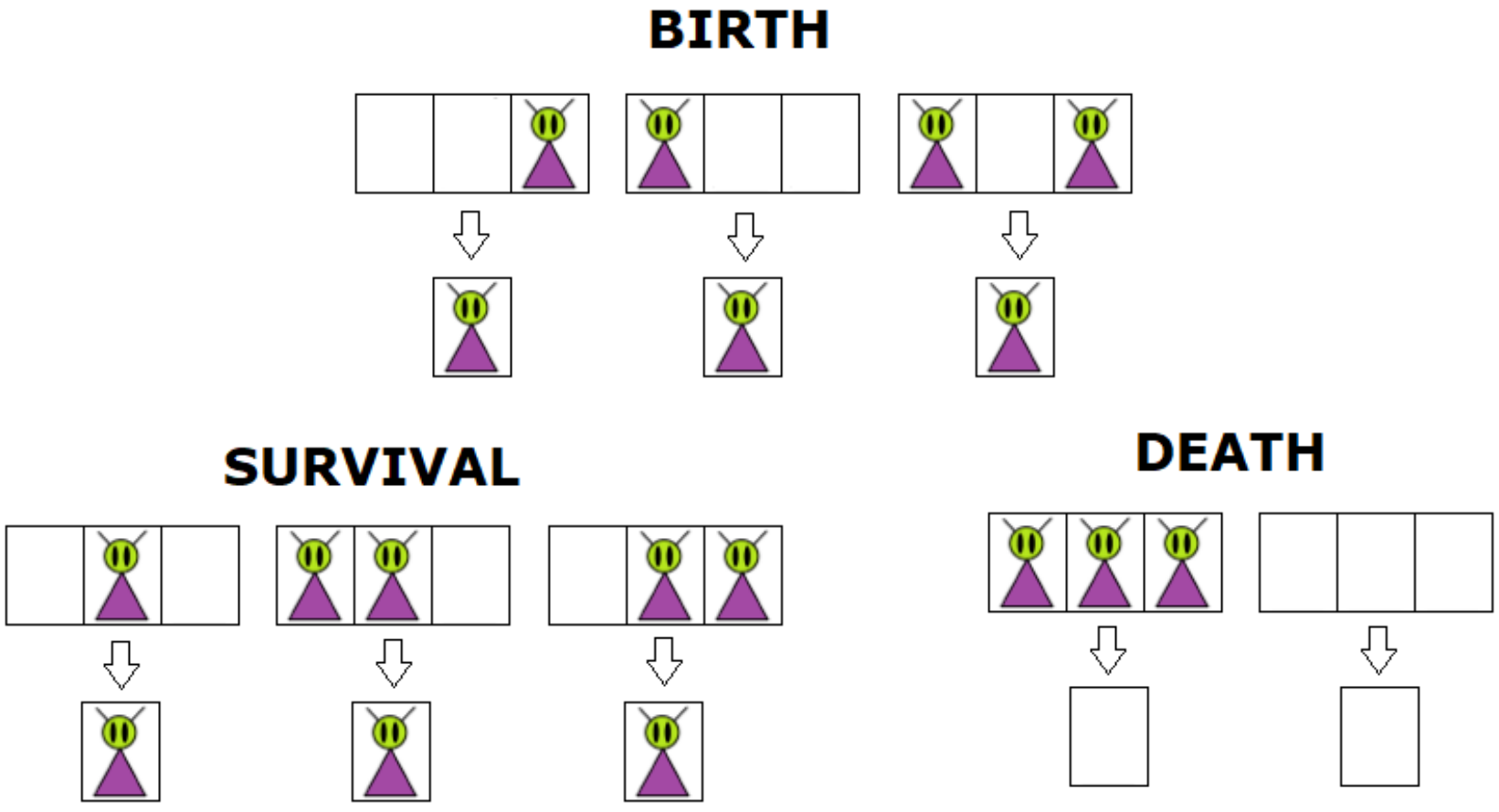
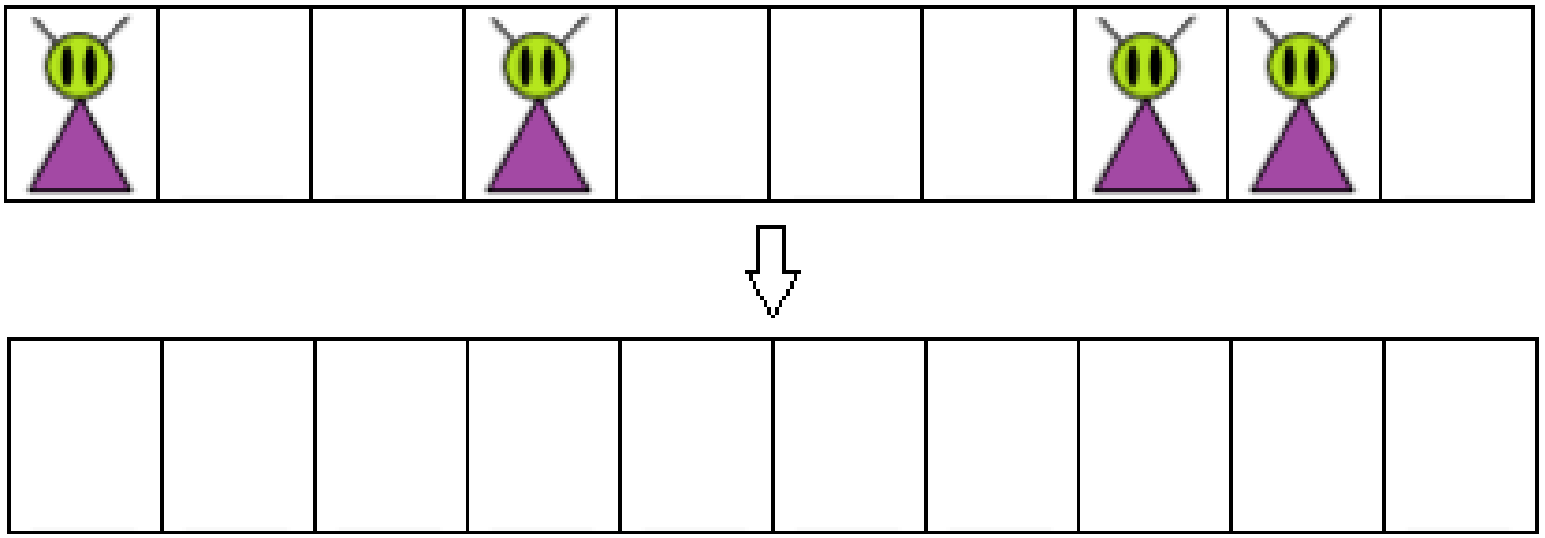


Example



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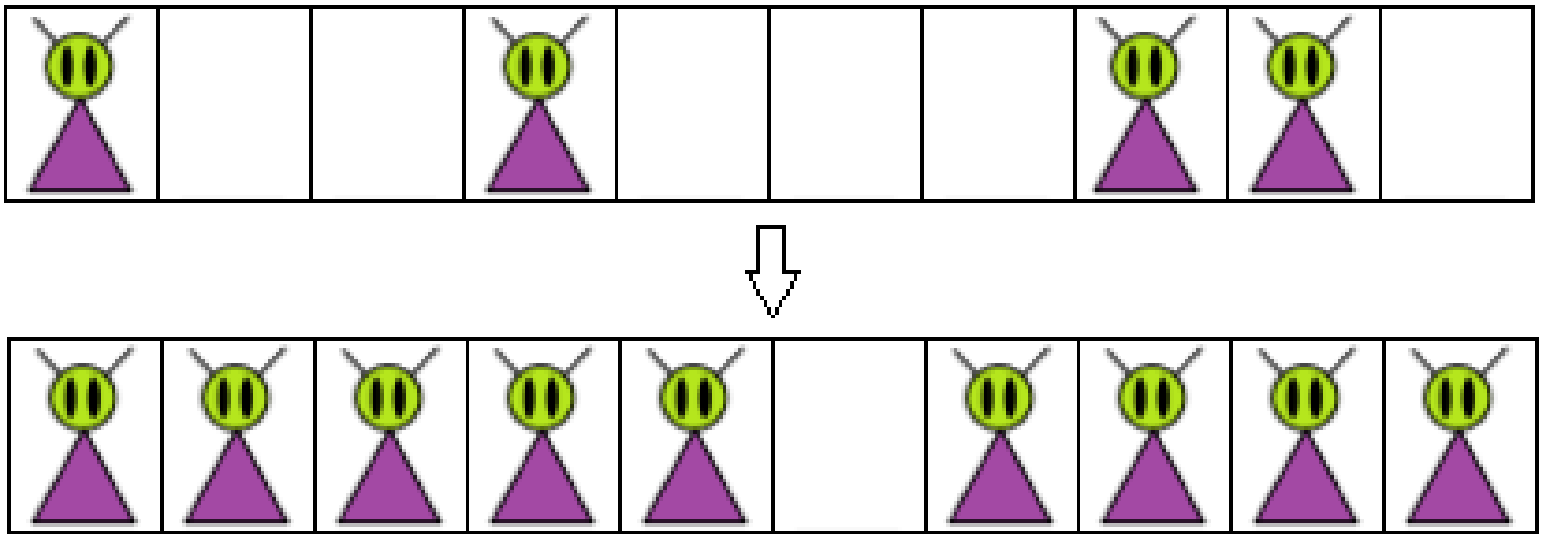
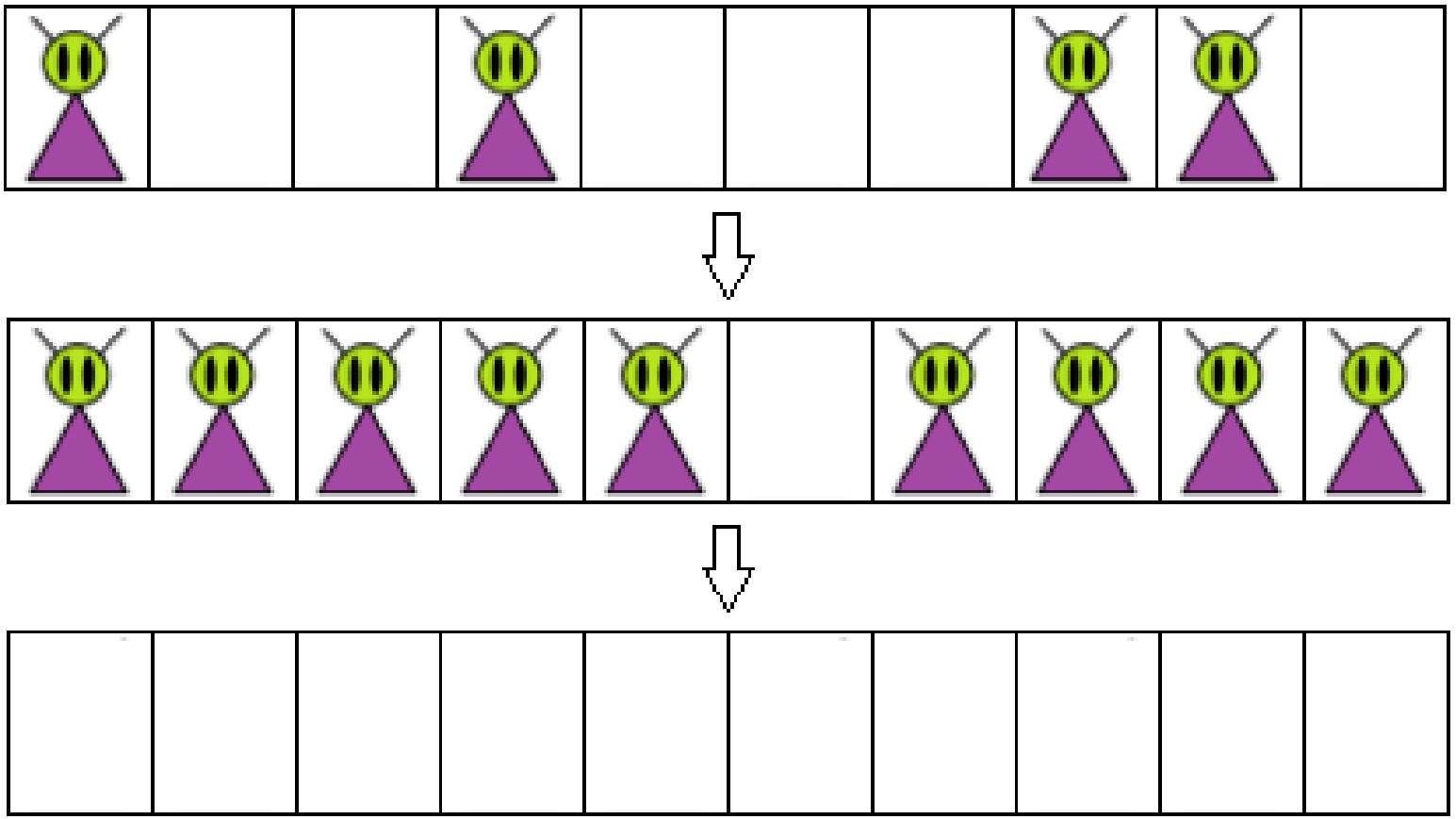
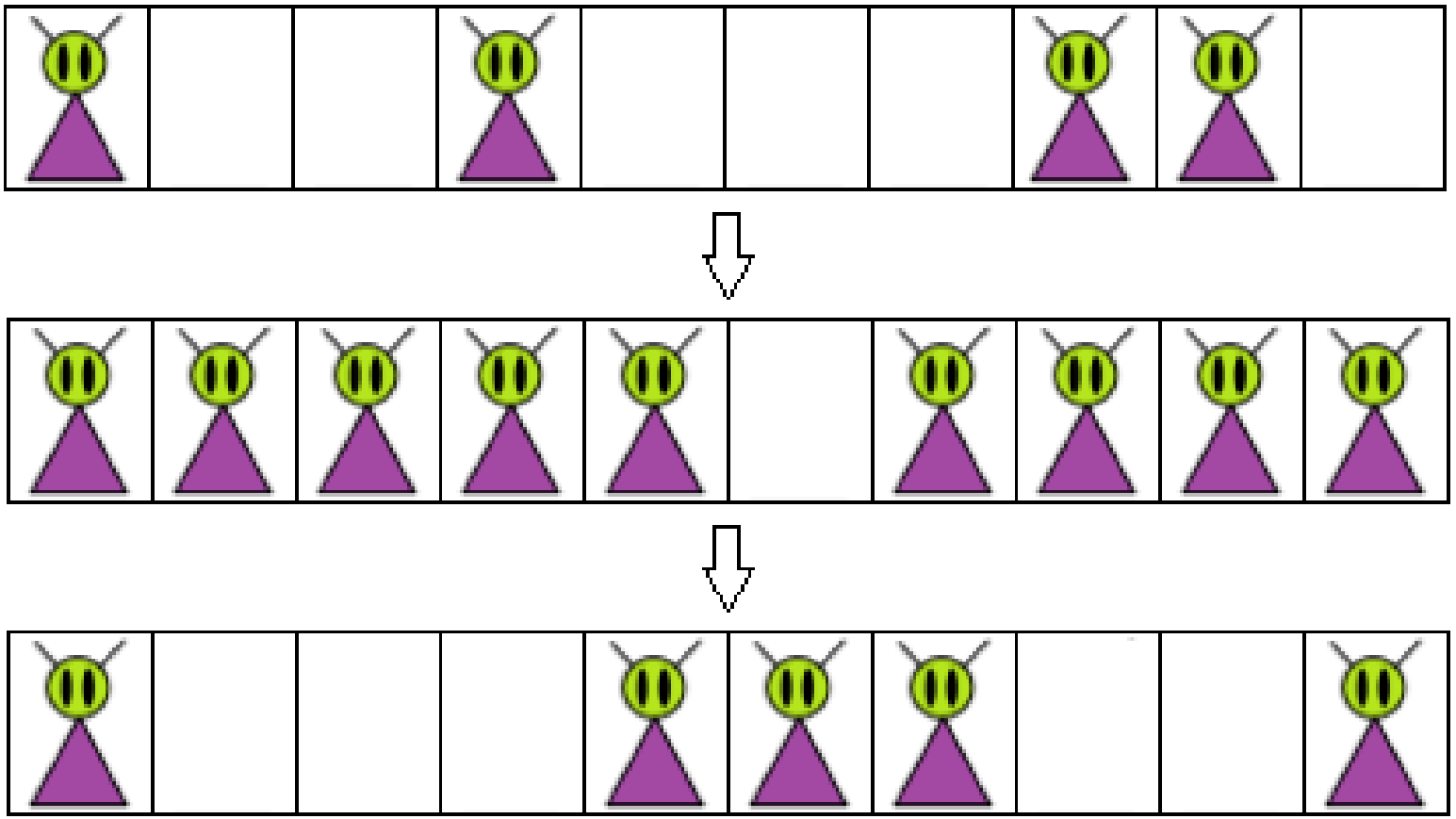


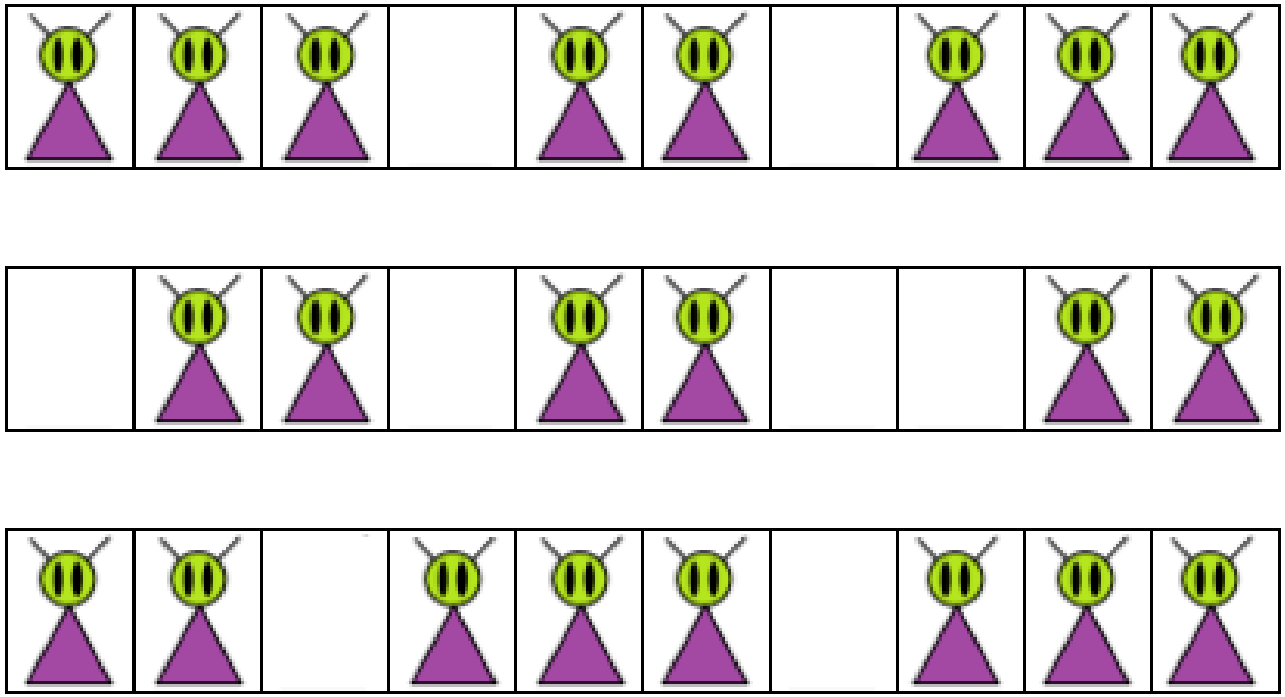
Example



Example



Exercise: According to the rules of friendification, which of the following arrangements of Budlings can occur?



What about different generation rules?

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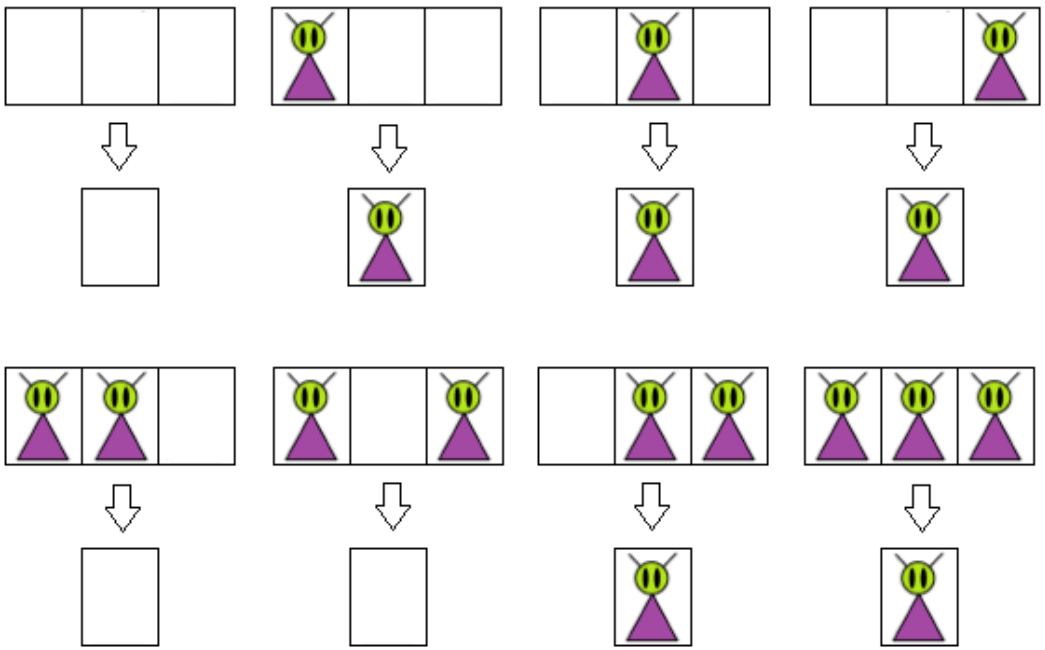
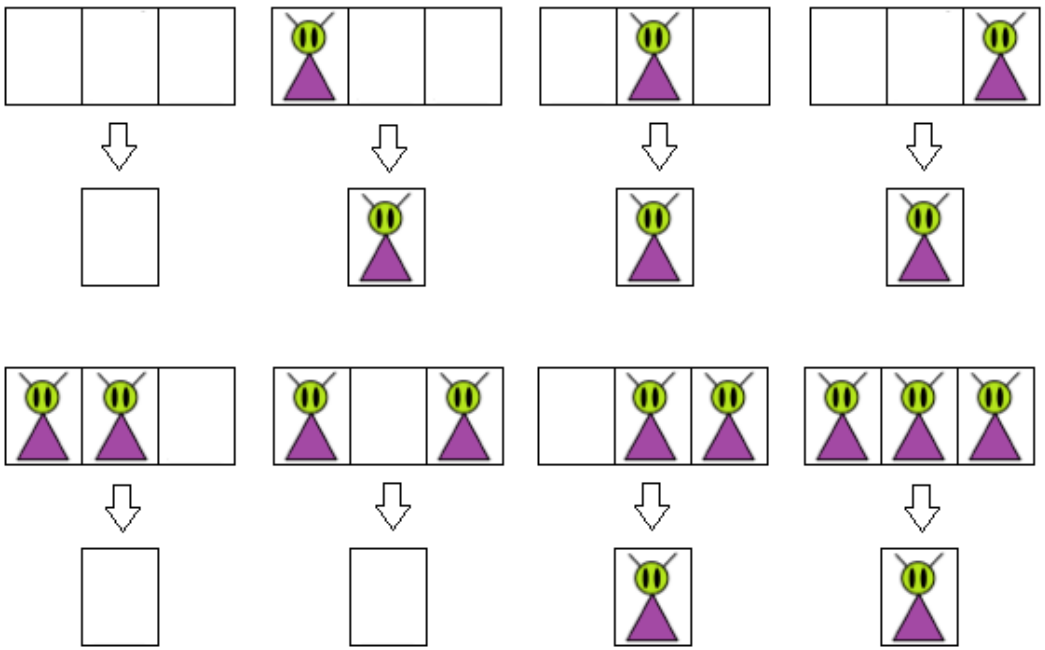
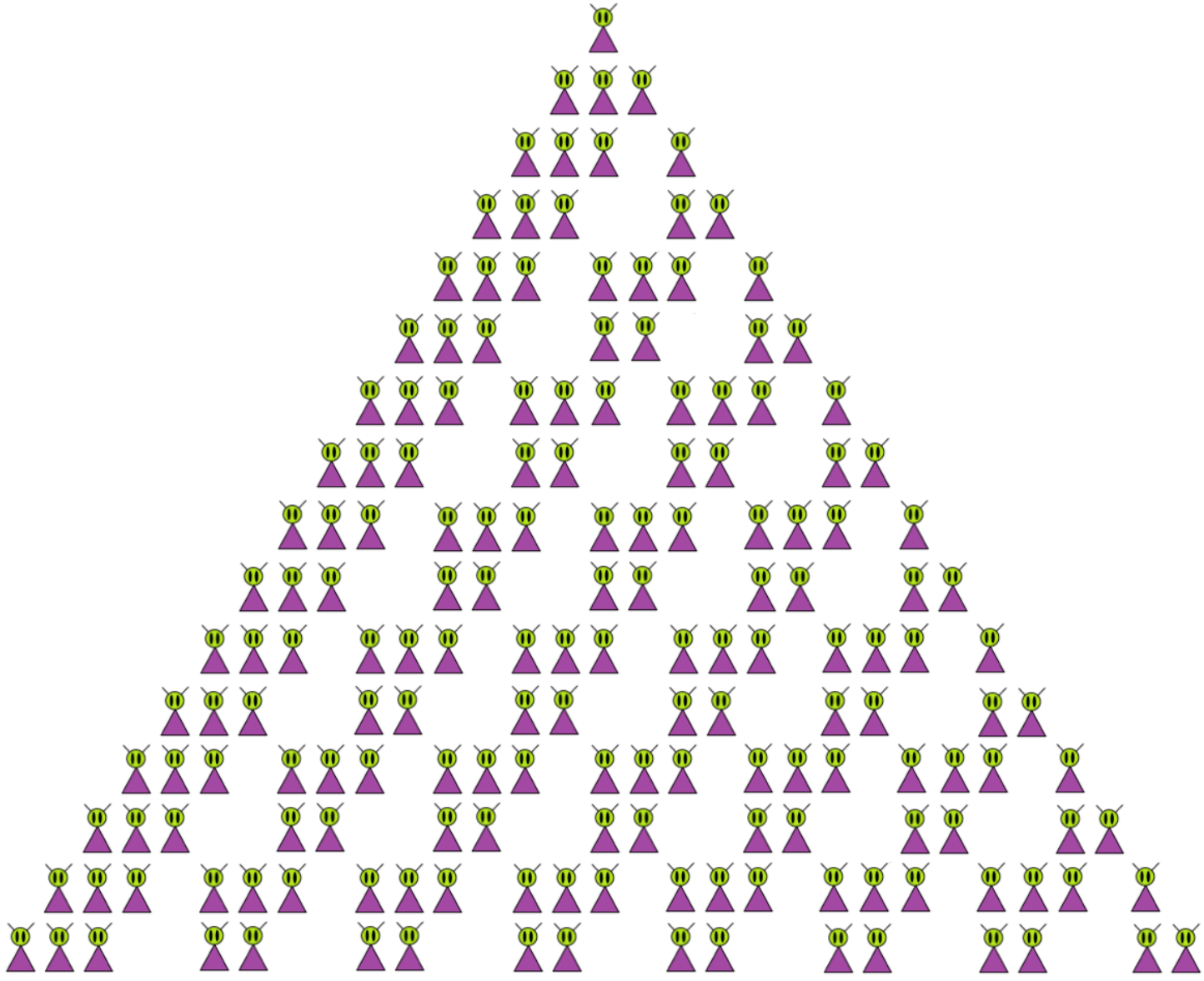


What about different generation rules?



Exercise: Starting with a single Budling, use these new rules to find their state after ten days.
(Don't just calculate — find the pattern!)



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It's easier to see what's going on if we use colors.

- Live alien $\rightarrow$ Black cell
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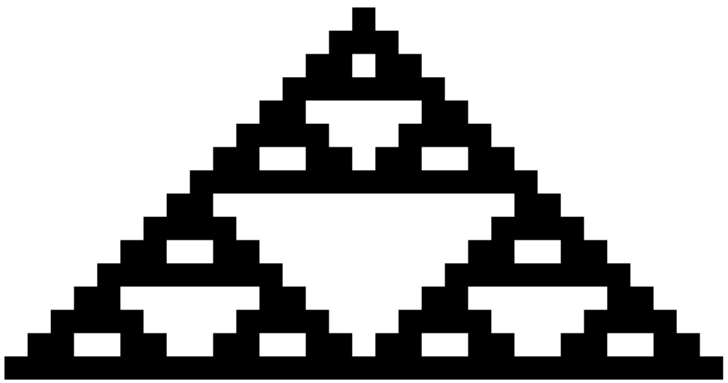

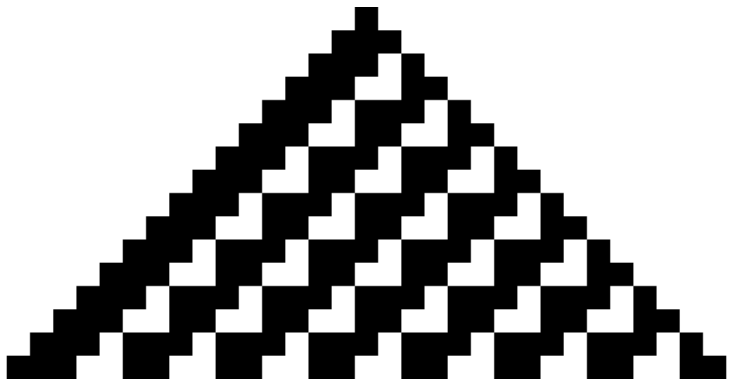

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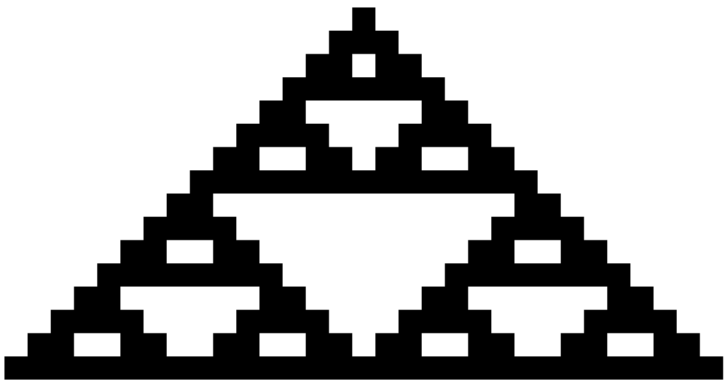

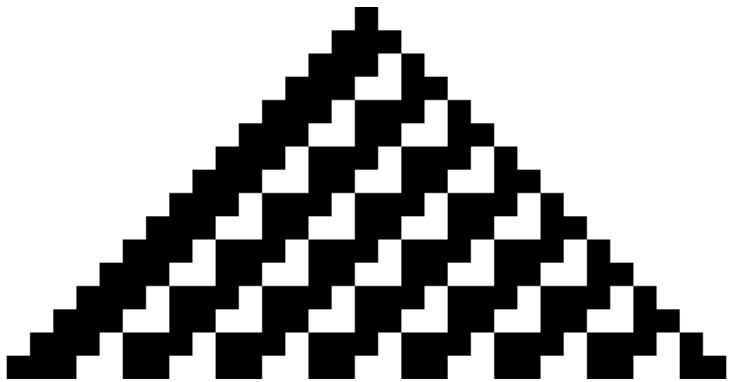

These patterns of cells are called **cellular automata**.

Cellular Automata

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A **cellular automaton** starts at an initial state of live and dead cells, and uses *transition rules* to create a new state.

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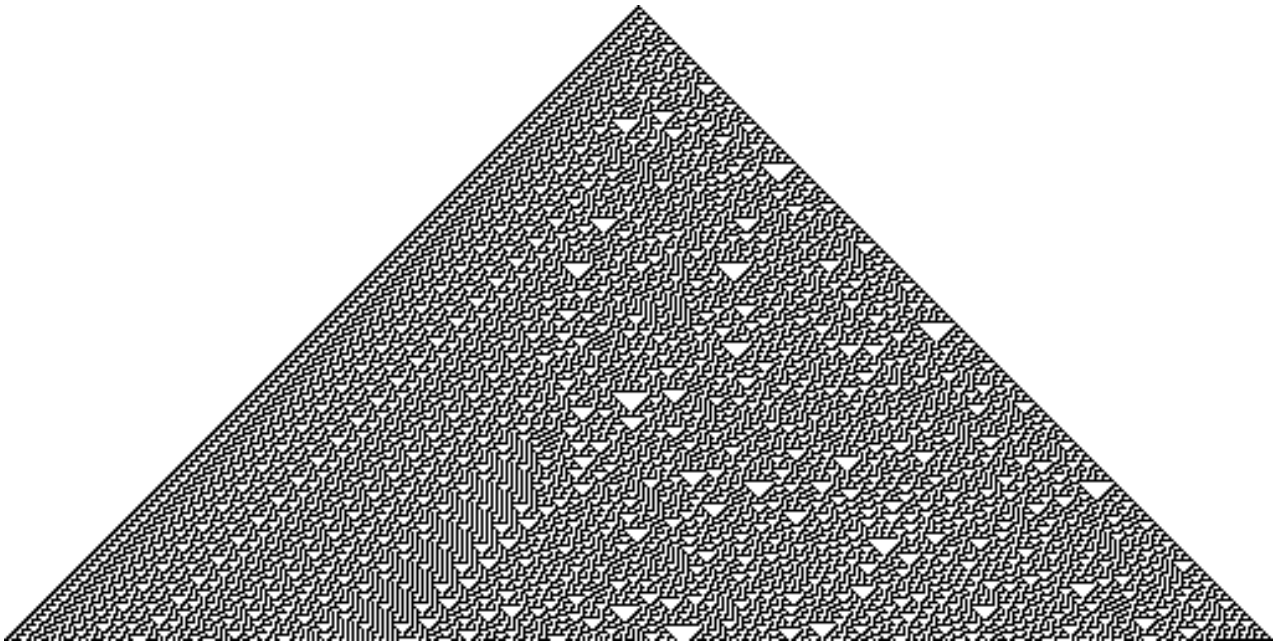

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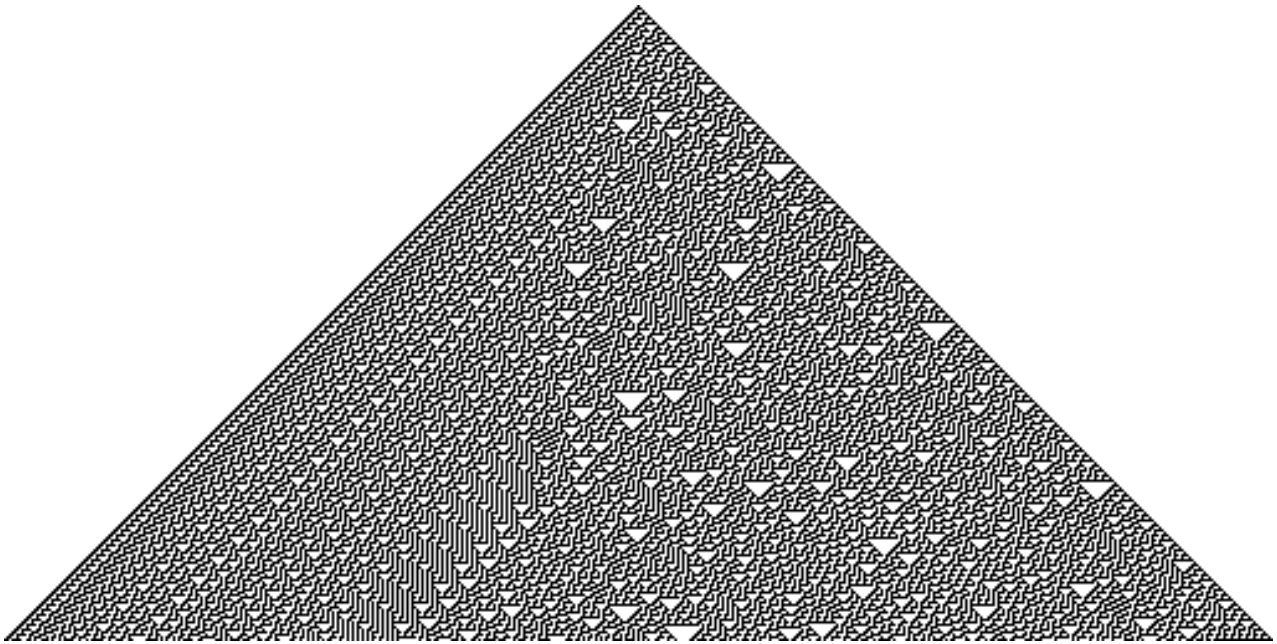


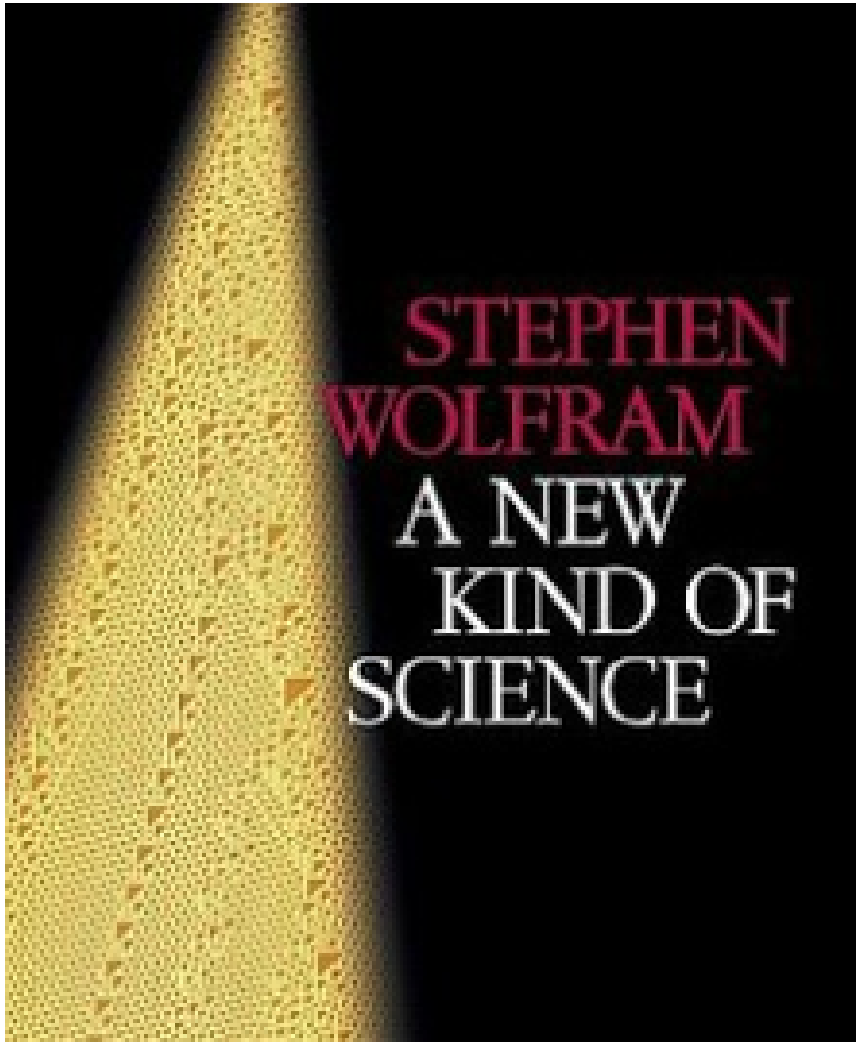
Even simple transition rules can generate chaotic patterns.

Stephen Wolfram

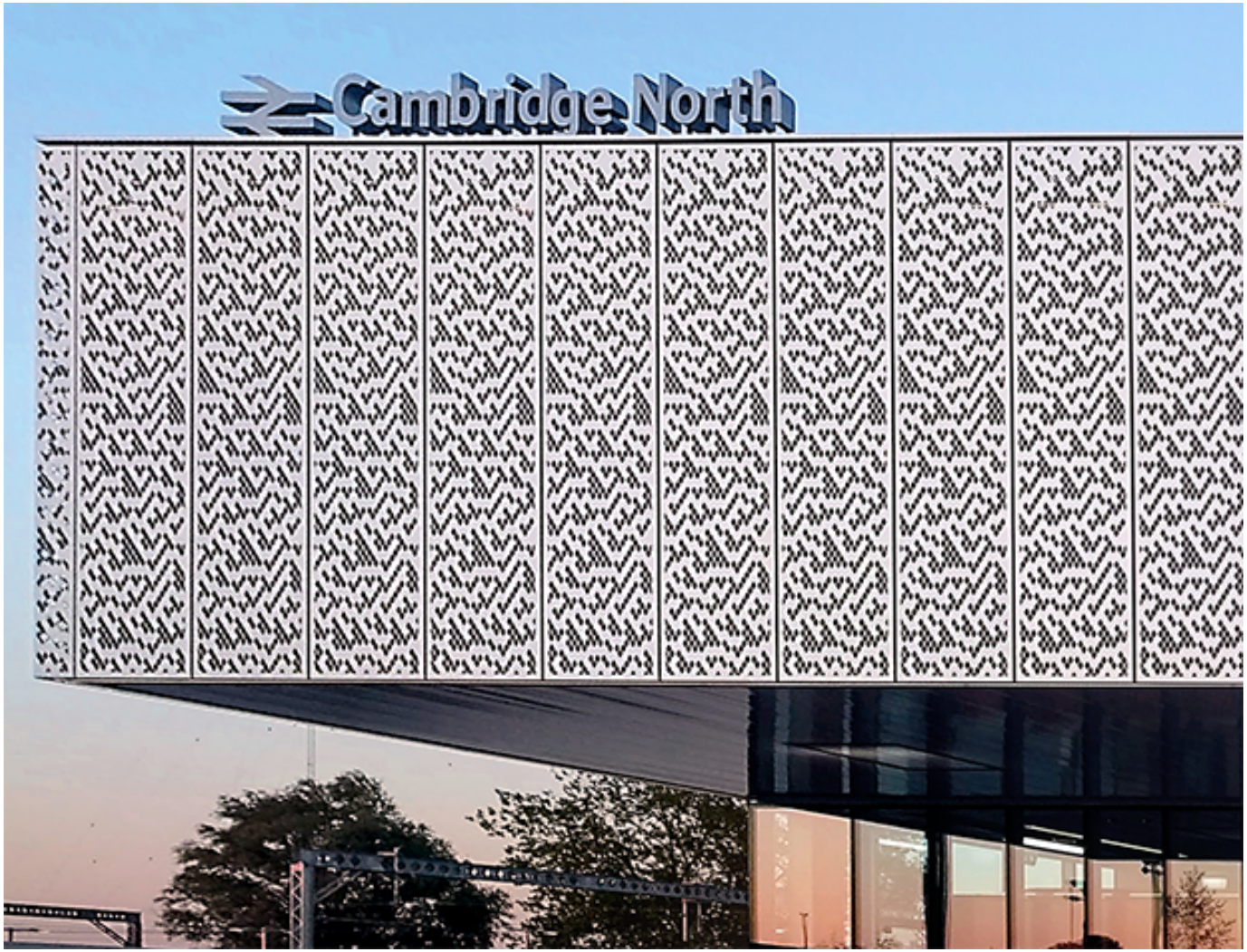
Creator of Mathematica and Elementary Cellular Automata
[VIDEO]

A New Kind Of Science

Free online: https://www.wolframscience.com/nks/



Automatic patterns in real life



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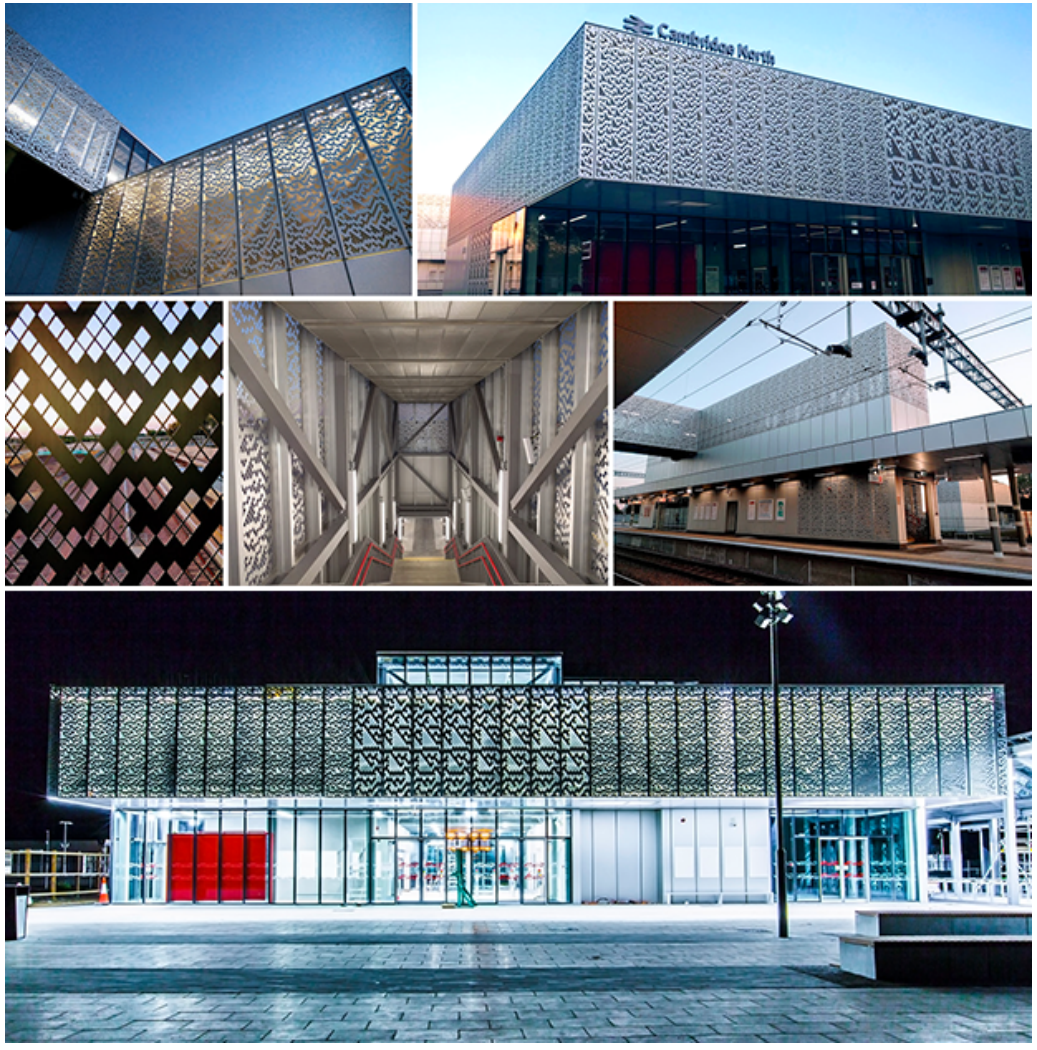

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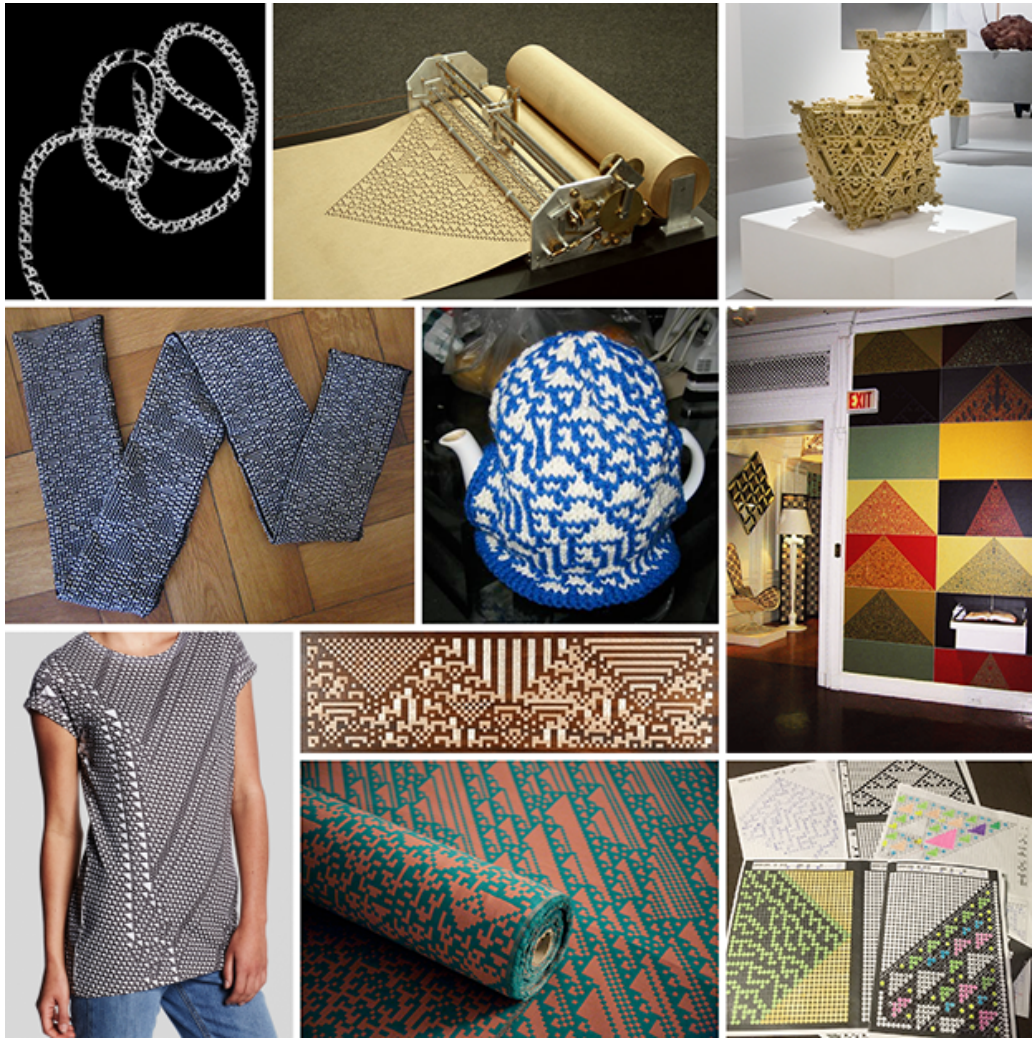

Rule 30: A closer look

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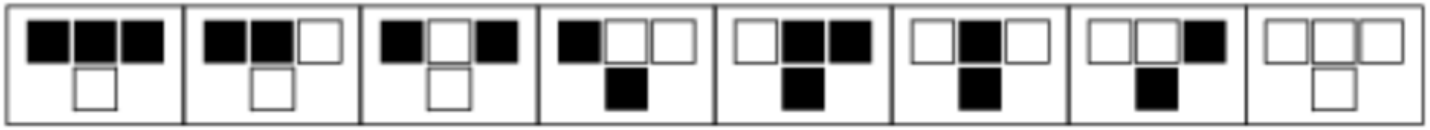

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This is an example of an **elementary cellular automaton**: two states (black/white), and the state of each cell only depends on the cells to its left and right.

Rule 30: A closer look

This is an example of an **elementary cellular automaton**: two states (black/white), and the state of each cell only depends on the cells to its left and right.

Exercise: Iterate Rule 30 ten times, starting with just one black cell.

Fun With Programming

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It's probably easier to get a computer to do this.

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CellularAutomaton[30, {{1}, 0}, 40]
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Try different rules and initial states!

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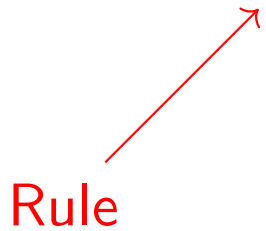

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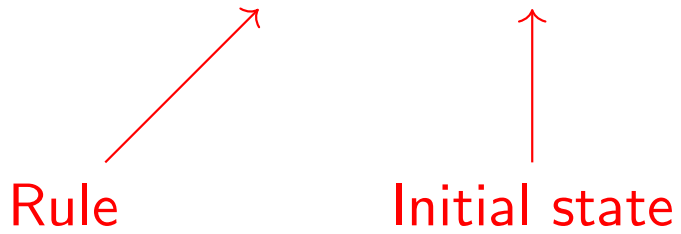

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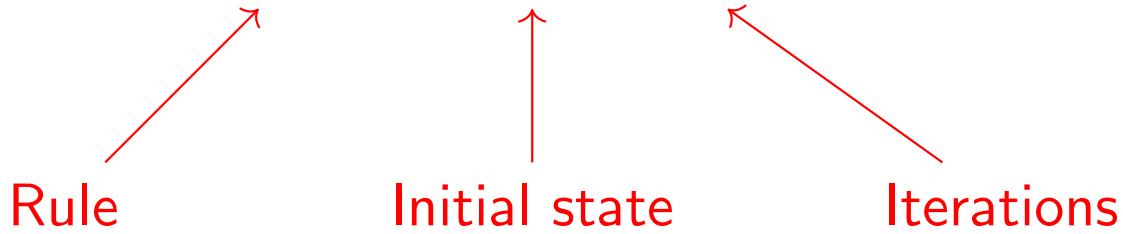

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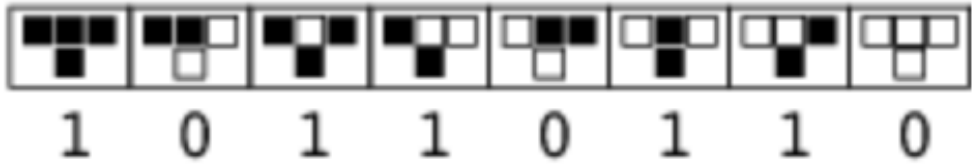
Why is it called “Rule 30”?



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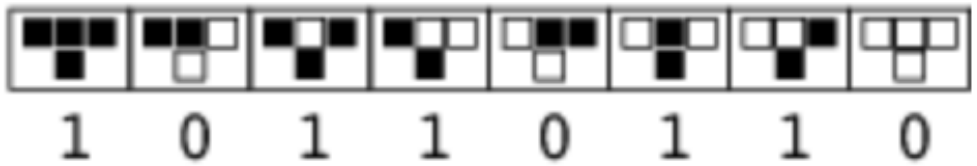

Wolfram calls this Rule 182 because 10110110 is the binary representation of the number 182.

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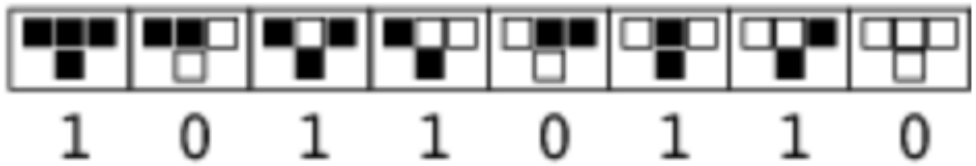

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$$(\mathbf{1})2^7+(\mathbf{0})2^6+(\mathbf{1})2^5+(\mathbf{1})2^4+(\mathbf{0})2^3+(\mathbf{1})2^2+(\mathbf{1})2^1+(\mathbf{0})2^0=182.$$

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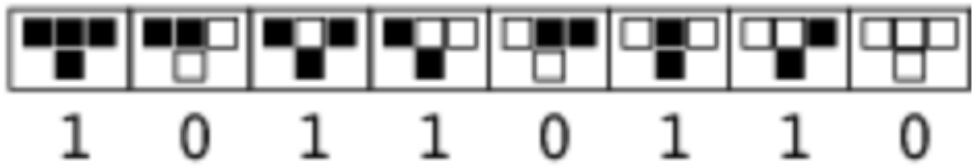

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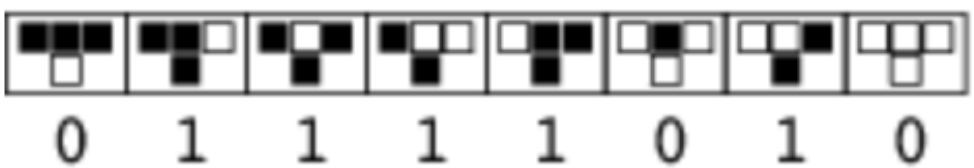
Another example: the binary expansion of 122 is 1111010, so Rule 122 is



Exercises

- Find the binary expansion of 90, and use it to write down the transition rules for Rule 90.
- The following transition rules represent which rule number?

- How many elementary cellular automata are there in total?

Solutions

- Find the binary expansion of 90, and use it to write down the transition rules for Rule 90.

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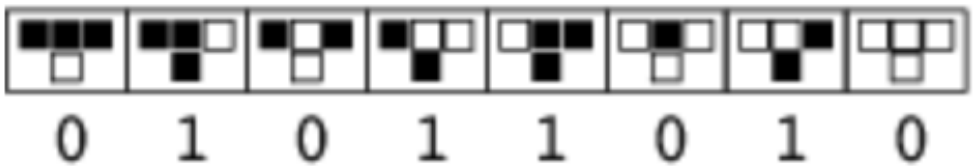

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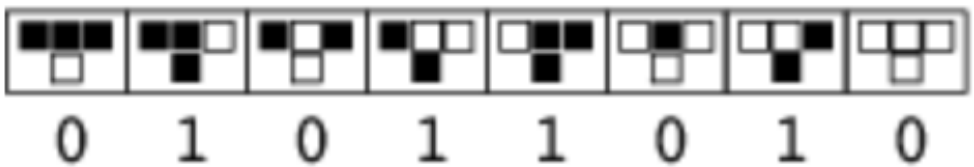

- Which cellular automaton has the following transition rules?

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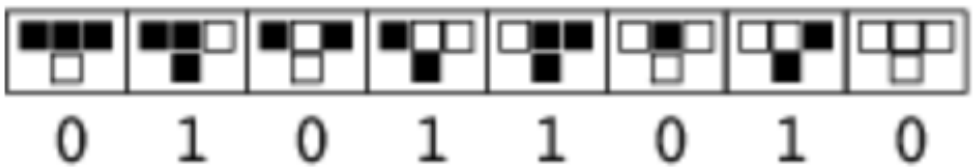

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  Solution: $10111110 = 190$, so this is Rule 190.

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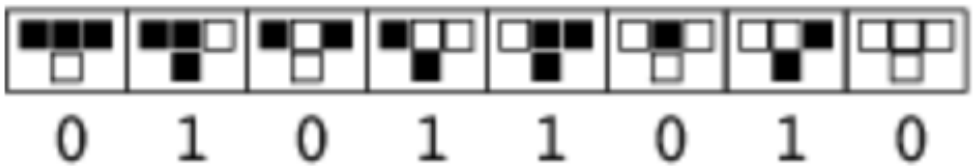

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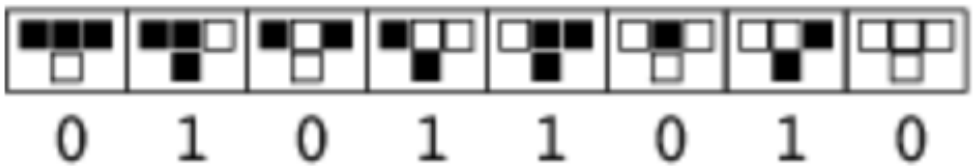

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- How many elementary cellular automata are there in total?

  Solution: There are $2^8 = 256$!

Changing the initial state

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So far we have started with just a single live cell.

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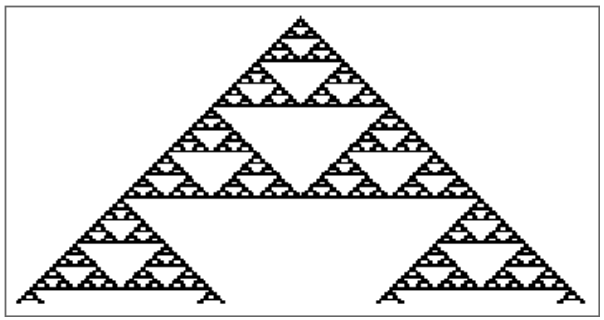

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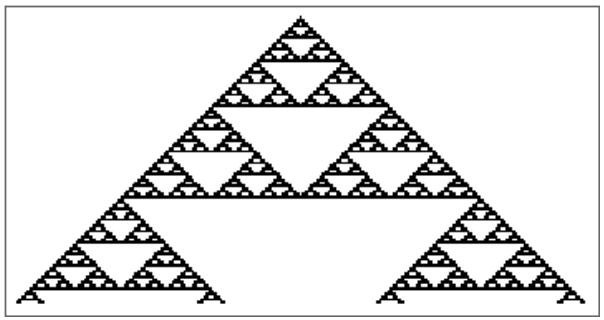
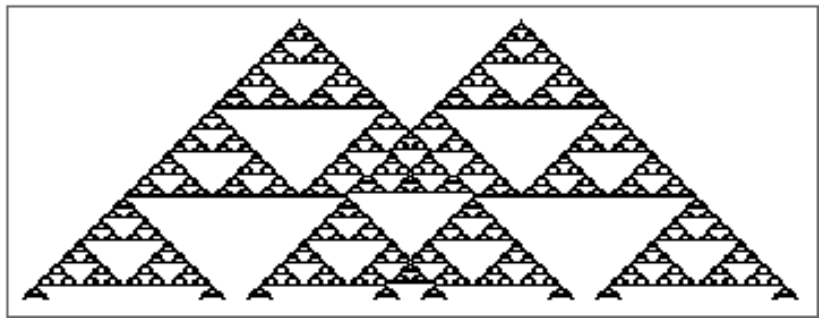


Two live states



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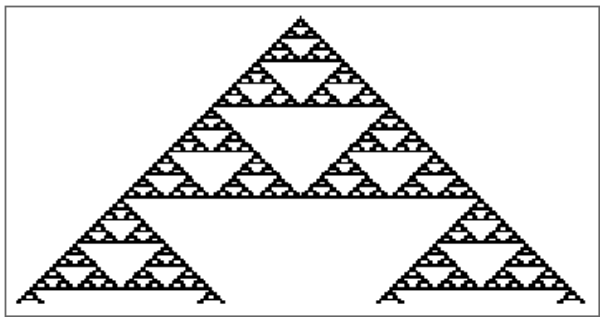

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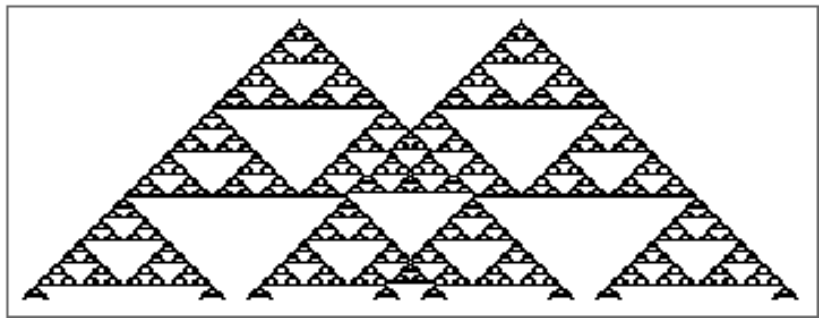
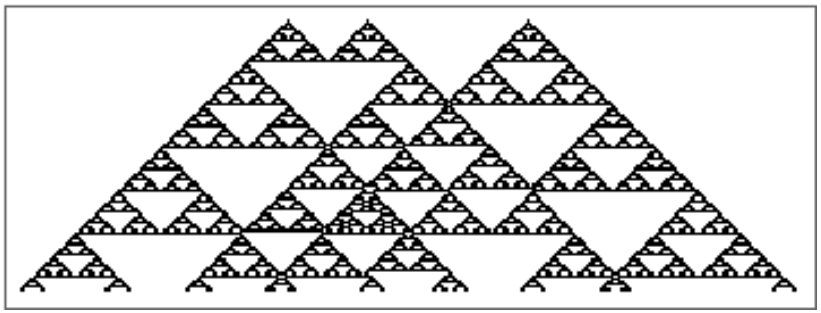


Three live states



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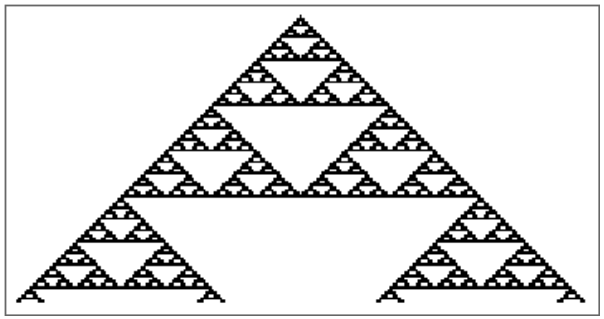

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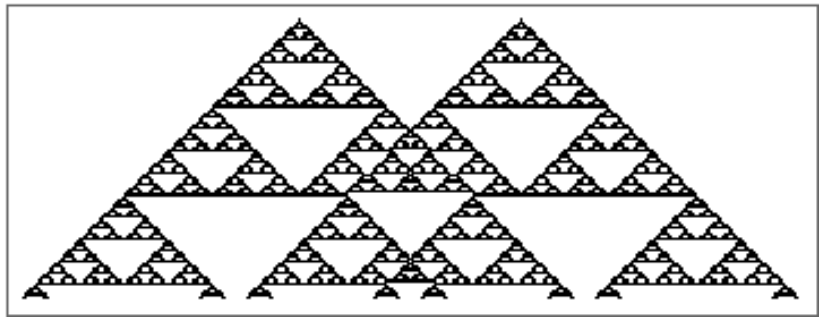

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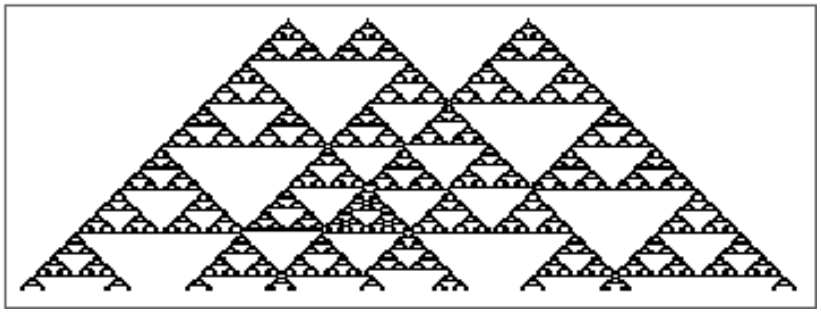
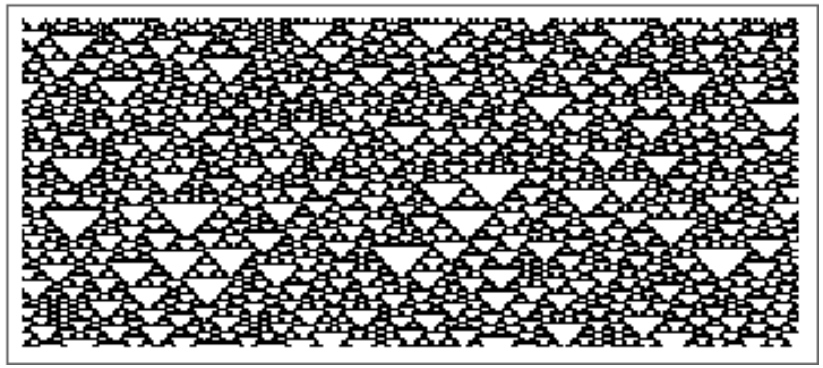


Random



Changing the initial state

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- Single live cell:

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```
CellularAutomaton[173, {{1},0}, 150]
```

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  $\Longrightarrow$ Runs Rule 173 with 150 iterations, starting with initial state $\cdots 0001000 \cdots$

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- Block padded with zeros:

Changing the initial state

- Single live cell:

  CellularAutomaton[173, {{1},0}, 150]

  $\Longrightarrow$ Runs Rule 173 with 150 iterations, starting with initial state $\cdots 0001000 \cdots$

- Block padded with zeros:

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Changing the initial state

- Single live cell:

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- Random state:

Changing the initial state

- Single live cell:

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  $\Longrightarrow$ Initial state is $\cdots 000101001000 \cdots$

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  RandomInteger[1, 250]

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- Single live cell:

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  $\Longrightarrow$ Runs Rule 173 with 150 iterations, starting with initial state $\cdots 0001000 \cdots$

- Block padded with zeros:

  CellularAutomaton[173, {{1,0,1,0,0,1},0}, 150]

  $\Longrightarrow$ Initial state is $\cdots 000101001000 \cdots$

- Random state:

  RandomInteger[1, 250]

  $\Longrightarrow$ Randomly generates a sequence of 250 zeros and ones.

Why Mathematicians Love Cellular Automata

So far, we have seen how to *generate* cellular automata using transition rules.

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But aside from making cool shower curtains, what's interesting about these?

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But aside from making cool shower curtains, what's interesting about these?

One of the most interesting subjects in cellular automata — with many unsolved problems today! — is **Gardens Of Eden**.

A Garden Of Eden in Rule 126

Iterate Rule 126 starting with a single live cell.

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Question: If you keep iterating Rule 126, will you ever reach a single live cell again?

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Or three? $\cdots 00010101000 \cdots$

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Or three? $\cdots 00010101000 \cdots$

Is there *any* initial state that will ever result in a single live state?

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Why not? We can *prove* it mathematically!

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What we just saw is that Rule 126 has a Garden Of Eden: a single live state!

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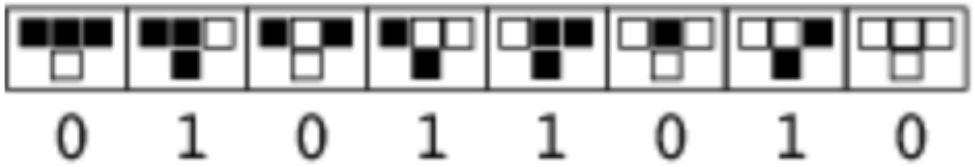

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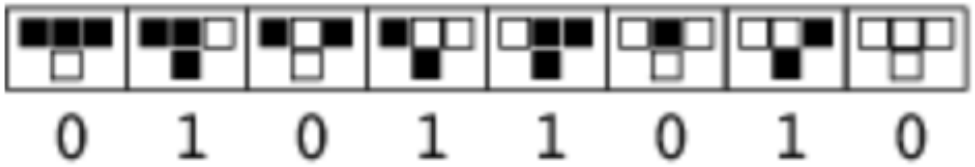

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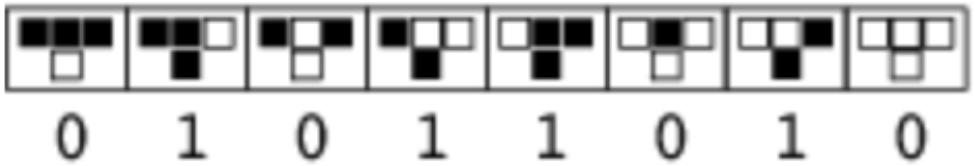

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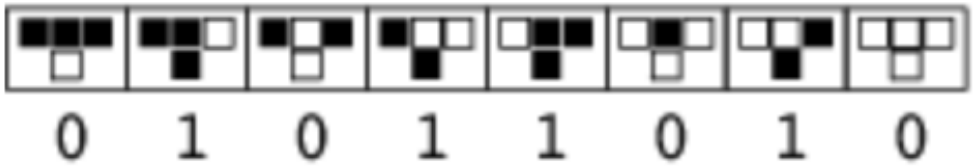

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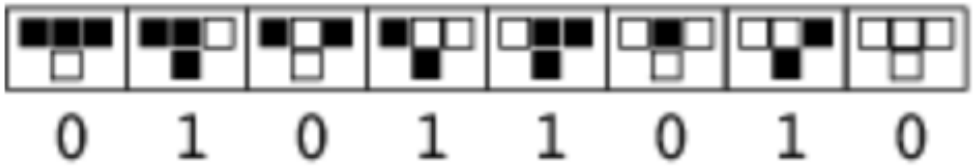

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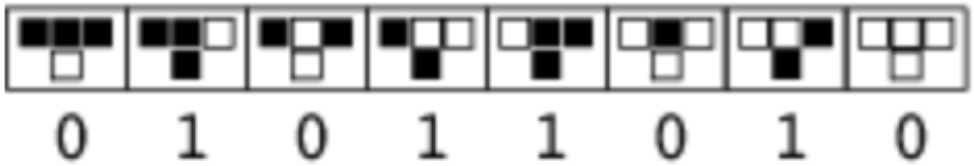

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What did you notice?

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Here is another pair of twin states for Rule 90: 10101 and 101000101.

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On the other hand, Rule 126 has a Garden Of Eden, so it **must** have twin states!

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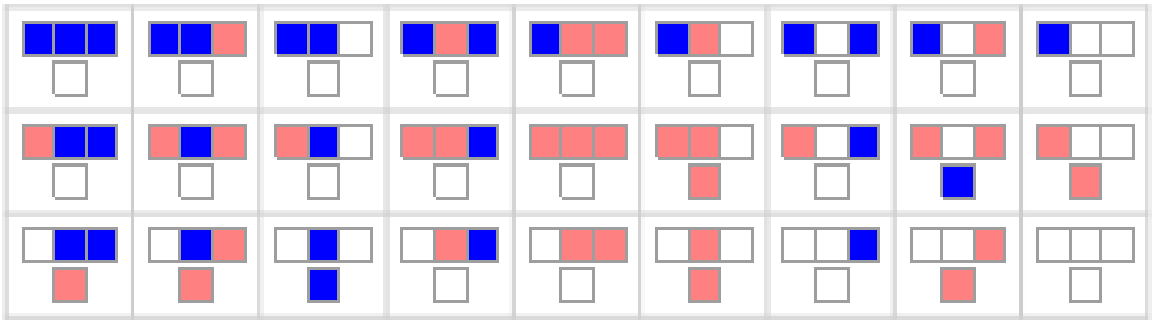

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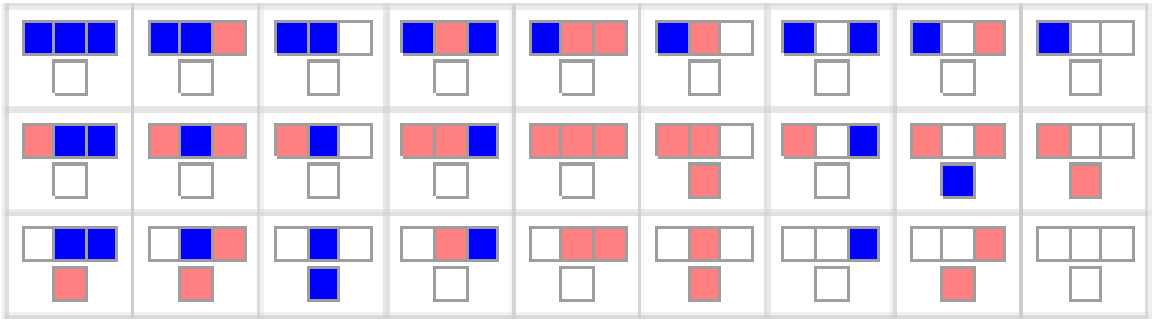

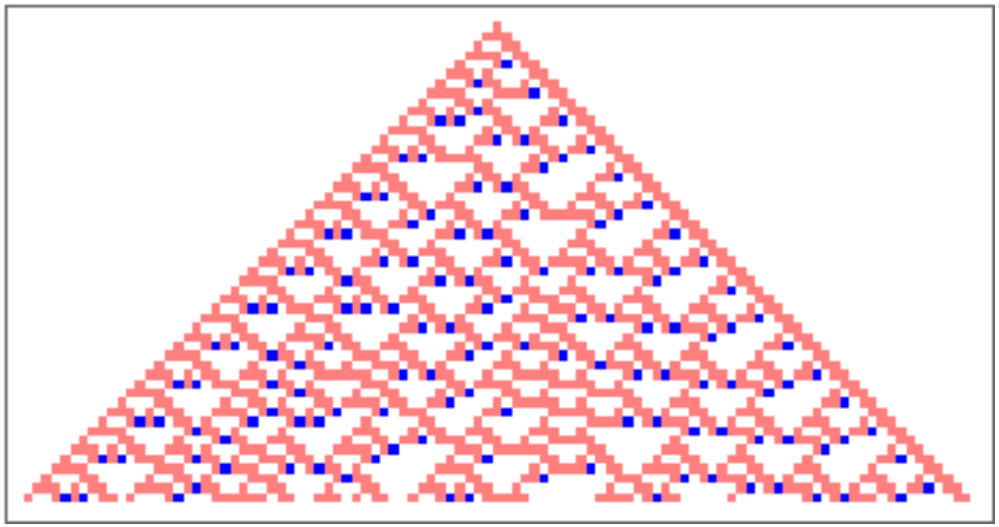

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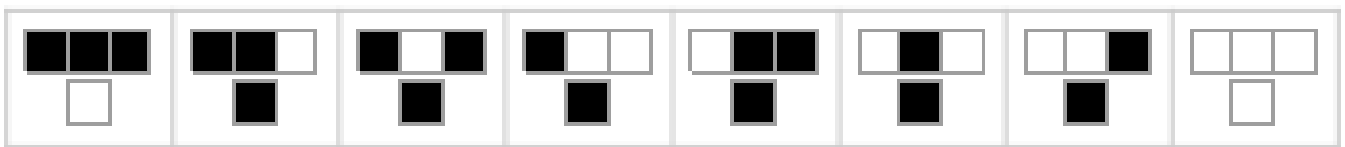

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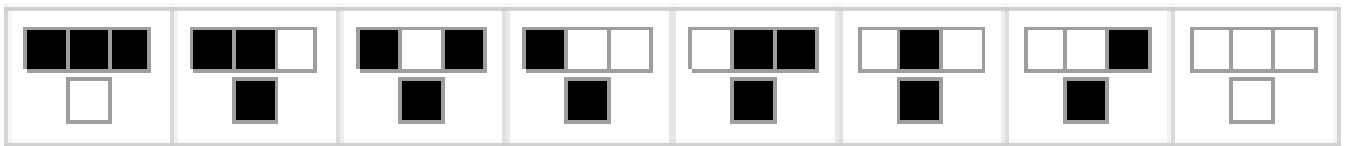

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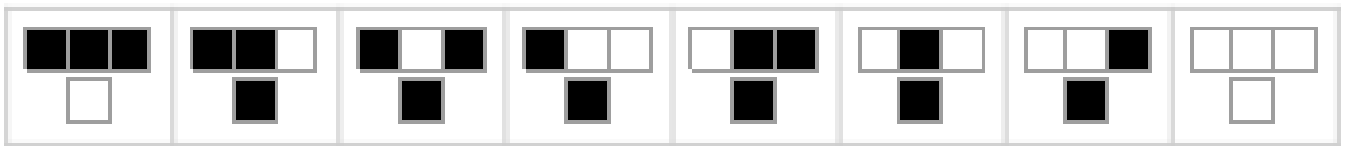

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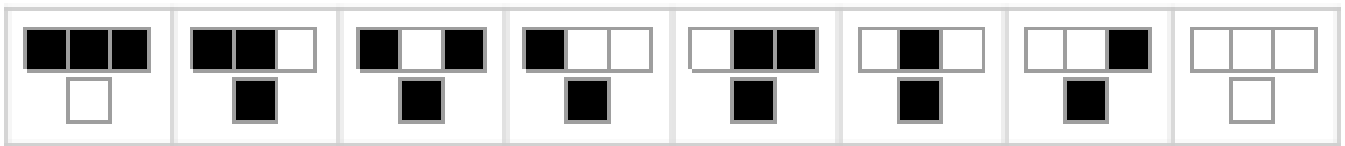

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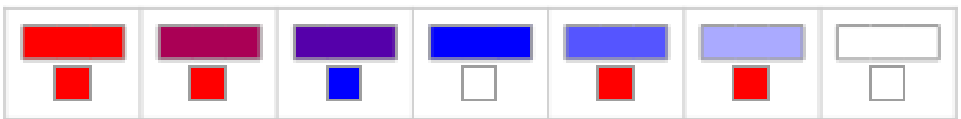

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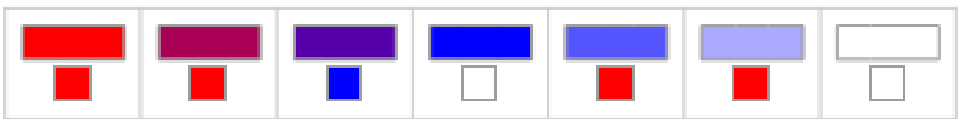

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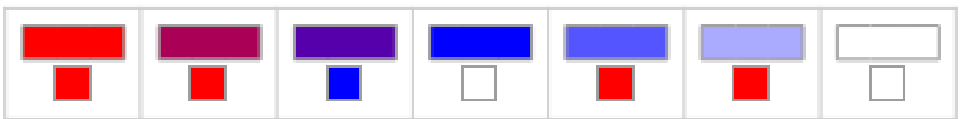

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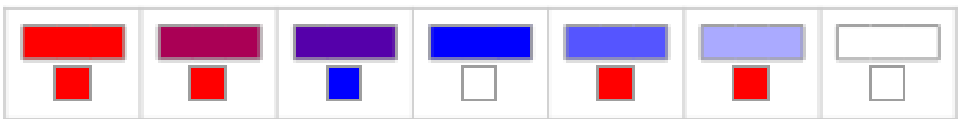

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Totalistic automata: A Code Challenge

Exercise: Using Wolfram Language, write code that generates a 4-color totalistic rule with initial state 102321.

See if you can find one that looks cool! Then use RulePlot to print out its transition rules.

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