Intermediate Math Circles
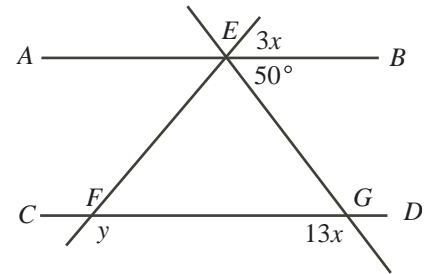# Wednesday 10 October 2018
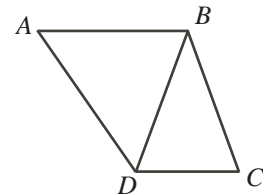# Problem Set 1

1. In the diagram, AB is parallel to CD. Determine the values of x and y.
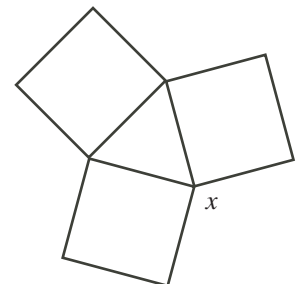
2. Triangle ABC has a right angle at B. AC is extended to D so that $CD = CB$. The bisector of angle A meets BD at E. Prove that $\angle AEB = 45°$.

3. In the diagram, AB is parallel to DC and $AB = BD = BC$. If $\angle A = 52°$, determine the measure of $\angle DBC$.
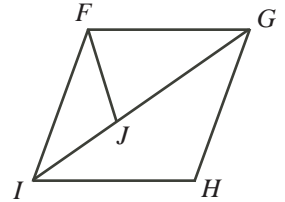
4. The diagram shows three squares of the same size. What is the value of x?

5. The diagram shows a rhombus $FGHI$ and an isosceles triangle FGJ in which $GF = GJ$. Angle FJI equals 111°. What is the measure of angle JFI?
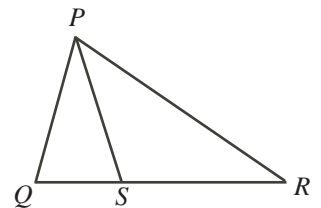
6. $ABCD$ is a square. The point E is outside the square so that CDE is an equilateral triangle. Determine the measure of angle BED.

7. The diagram shows two isosceles triangles in which the four angles marked x are equal. The two angles marked y are also equal. Find an equation relating x and y.
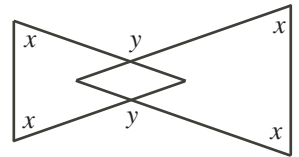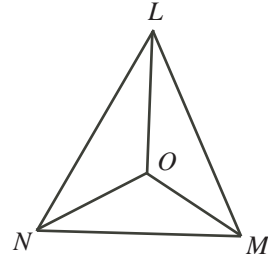
8. In the diagram, QSR is a straight line. $\angle QPS = 12°$ and $PQ = PS = RS$. What is the measure of $\angle QPR$?
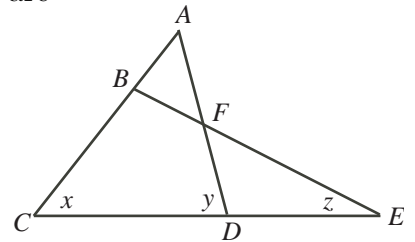
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9. The three angle bisectors of triangle LMN meet at a point O as shown. Angle LNM is 68°. What is the size of angle LOM?
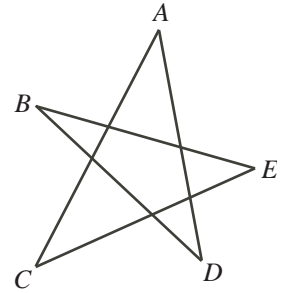
10. In the figure shown, $AB = AF$ and ABC, AFD, BFE, and CDE are all straight lines. Determine an equation relating x, y and z.

11. What is the measure of the angle formed by the hands of a clock at 9:10?
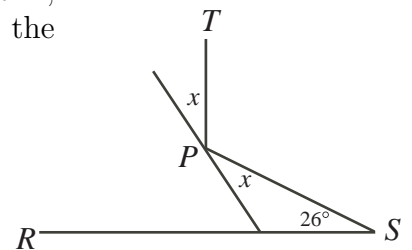
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12. Determine the sum of the angles A, B, C, D, and E in the five-pointed star shown.

13. In $\triangle PQR$, $PQ = PR$. PQ is extended to S so that $QS = QR$.
    Prove that $\angle PRS = 3(\angle QSR)$.

14. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. What is the value of x?
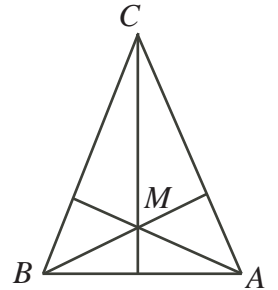
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15. In the diagram, PW is parallel to QX, S and T lie on QX, and U and V are the points of intersection of PW with SR and TR, respectively. If $\angle SUV = 120°$ and $\angle VTX = 112°$, what is the measure of $\angle URV$?
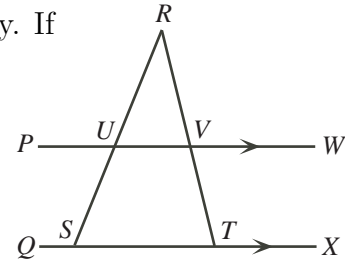
16. In the diagram, let M be the point of intersection of the three altitudes of triangle ABC. If $AB = CM$, then what is $\angle BCA$ in degrees?

17. The diagram shows a regular nonagon with two sides extended to meet at point X. What is the size of the acute angle at X?
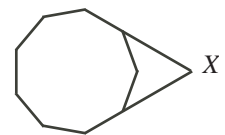
18. The angles of a nonagon are nine consecutive numbers. What are these numbers?

19. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram, $TREND$ is a regular pentagon, PEA is an equilateral triangle, and $OPEN$ is a square. Determine the size of $\angle EAR$.
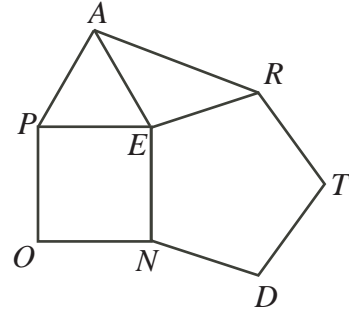
20. Three regular polygons meet at a point and do not overlap. One has 3 sides and one has 42 sides. How many sides does the third polygon have? Can you find other sets of three polygons that have this property?

Answers

1. $x = 10°,\ y = 150°$ 3. $\angle DBC = 28°$ 4. $x = 120°$

5. $\angle JFI = 27°$ 6. $\angle BED = 45°$ 7. $y = 2x$

8. $\angle QPR = 54°$ 9. $\angle LOM = 124°$ 10. $x - y + 2z = 0°$

11.   145°   12.   180°   14. $x = 32°$

15. $\angle URV = 52°$ 16. $\angle BCA = 45°$ 17.   60°

18.   136° to 144°   19. $\angle EAR = 39°$ 20.   7 sides

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