

## Egyptian Mathematics Problem Set

- (i) Use the Egyptian area of a circle  $A = (8d/9)^2$  to compute the areas of the following circles with given diameter
1.  $d = 1$
  2.  $d = 3$
  3.  $d = 9/2$
- (ii) Based on the tax rate in the lecture, compute the amount of grain needed to be taxed on the following plots of land
1. A plot of 10 setats.
  2. A plot of four khet by two khet.
  3. A circular plot of diameter 18 khet (Hint: refer back to question 1!)
- (iii) (Programming) Write a program that determines what day of the year it is given the Egyptian calendar of alternating months of 29 and 30 days. Can you factor in a leap year if a leap year is to happen only in years divisible by 5?
- (iv) Using Fibonacci's Greedy Algorithm from the notes, write each of the following fractions as a sum of unit fractions
1.  $4/7$
  2.  $5/7$
  3.  $5/17$
  4.  $9/11$
- (v) The Egyptians tried really hard to make the pyramids in the inverse ratio of the golden ratio. How well did they do? The actual measurements of the pyramids are 481.2ft high and 377.89ft for half the base length (the value we called  $b$ ). Compute the value of the slant length  $a$  and compare  $b/a$  to the reciprocal of the golden ratio.
- (vi) Calculate the volume of the Giza pyramid from the previous problem pyramid with dimensions. If a future Pharaoh wanted to create a pyramid that was 50% larger in size but still had  $b/a$  in ratio with the reciprocal of the golden ratio, what would the half base and height of such a pyramid be?
- (vii) Calculate the volume of a frustum with height 4, lower square base length of 5 and upper square base length of 3. How would such a formula look for a cone instead of a square based pyramid?
- (viii) Add 432 and 199 like an Egyptian! What difficulties to you notice from the Egyptian system?
- (ix) Write the fraction  $11/17$  as the sum of 5 distinct unit fractions. (Hint: What happens when you add  $\frac{1}{n(n+1)}$  and  $\frac{1}{n+1}$ )?

(x) Show that for all positive integers  $n > 2$ , the fraction  $\frac{2}{n}$  can be written as a sum of two non-zero unit fractions with different denominators.

(xi) 1. Suppose  $a$  is a positive divisor of  $n^2$ . Let  $b = n^2/a$ . Show that

$$\frac{1}{n} = \frac{1}{n+a} + \frac{1}{n+b}$$

2. Prove that for  $m, n \in \mathbb{N}$  coprime with  $2m \leq 3n$ , the equation

$$\frac{m}{n} = \frac{1}{x} + \frac{1}{y}$$

with  $x, y \in \mathbb{N}$  holds if and only if there exists a positive divisor  $a$  of  $n^2$  such that  $m \mid (n+a)$ ,  $m \mid \left(n + \frac{n^2}{a}\right)$  and that  $x = \frac{n+a}{m}$  and  $y = \frac{n+n^2/a}{m}$  (or vice versa).

3. Prove that for all  $n \in \mathbb{N}$ ,  $3/n$  has a representation as a sum of two different unit fractions if and only if there exists a prime factor of  $n$  not congruent to 1 modulo 6.

4. Show that  $3/7$  can't be expressed as a sum of two unit fractions with different denominators.