Intermediate Math Circles  
Wednesday, March 29, 2017  
Problem Set 7

1. Using GeoGebra(geogebra.org), determine the locus of points that are twice as far from point A as they are from point B.

![Diagram of two circles intersecting]

Steps:

i. Construct and label two points A and B.

ii. Construct a line segment of arbitrary length. Label the end points M and N.

iii. Construct a circle with centre A and radius $MN$.  
Note: Can do this using the Input: bar and the command Circle[<Point>,<Radius Number>].

iv. Construct a circle with centre B and radius twice the length of MN.

v. Select the points of intersection of the two circles and label them C and D.  
Note: You may need to adjust the length of line segment MN so that the circles intersect.

vi. Right click on points C and D and select Trace On.

vii. Vary the length of line segment MN.

Questions:

(a) Describe the locus

(b) Change the location of point A. Describe how the locus changes
   i. when points A and B are closer together
   ii. when points A and B are farther apart

2. Using GeoGebra(geogebra.org), consider chords of equal length drawn in a circle. Determine the locus of the midpoints of the chords.
Steps:

i Construct a line segment MN. This will be the length of the chord.

ii Construct a circle with centre $A$ and point $P$.
   Hint: the command \texttt{Circle[<Point>,<Point>]} will be helpful

iii Construct a circle with centre $P$ and radius of length $MN$.
   Hint: remember command \texttt{Circle[<Point>,<Radius Number>]}]

iv Call the intersections of your two circles $Q_1$ and $Q_2$.
   Note: You can hide your recently created circle by right clicking on the circle and
   unselecting \textit{Show Object} and \textit{Show Label}.

v Using the line segment command create cords $PQ_1$ and $PQ_2$.

vi Construct the midpoints of line segments $PQ_1$ and $PQ_2$. Rename the midpoints $M_1$ and $M_2$.

vii Right click on points $M_1$ and $M_2$ and select \textit{Trace On}.

viii Vary the length of line segment MN.

Questions:

(a) Describe the locus of midpoints of the chords

(b) Where is do you suspect the centre of the locus is located?

(c) How would the locus change if you only had one of $M_1$ and $M_2$?

3. Given the points $A(2, 0)$ and $B(5, 0)$, find the equation of the locus of points that are twice as far from point A as they are from point B.
4. Determine an equation for each for the following circles
   (a) centre (0,0), through (−2, 3)
   (b) centre (0,0), x-intercepts at ±8
   (c) centre (3,4), through (0,0)
   (d) centre (−1,3), through (1,−1)
   (e) centre (−2,−2), y-intercept −2

5. (a) Show that the points P(−2, 4) and Q(2,−4) are both on the circle \(x^2 + y^2 = 20\).
   (b) Show that \(PQ\) is a diameter of the circle

6. Determine the equations of the circles with the given diameters
   (a) from (−3, 5) to (3,−5)
   (b) from (−1,2) to (5,8)

7. For the circle given by \(x^2 + y^2 = 34\),
   (a) show that the line segment from \(P(−5, 3)\) to \(Q(3, 5)\) is a chord of the circle;
   (b) find the midpoint M of the chord;
   (c) show that \(MO \perp PQ\)

8. A circle passes through the points \(A(−1,1)\) and \(B(6,0)\) and has its centre on the line \(x + 3y + 7 = 0\). Find the equation of the circle.