Intermediate Math Circles  
Wednesday, April 5, 2017  
Problem Set 8

1. Determine the coordinates of the vertices and foci for each of the following ellipses.

   (a) $x^2 + 9y^2 = 36$
   (b) $25x^2 + 16y^2 = 400$
   (c) $2x^2 + 3y^2 = 12$

2. A bubble over a set of tennis courts is semi-elliptical in cross-section, 20m wide at the ground, and 8 m high in the centre. How high is the bubble at a point 2m in from the outer edge?

3. A spacecraft is in a circular orbit 800 km above Earth. To transfer the craft to a lower circular orbit 150 km above Earth, the spacecraft must be placed in an elliptical orbit as shown with centre of Earth at one focus.

   Find an equation of the transfer orbit if the radius of the earth is 6336 km.

4. Determine the equation for each of the following parabolas.

   (a) vertex $(0, 0)$, focus $(0, 3)$
   (b) vertex $(0, 0)$, focus $(-2, 0)$
   (c) vertex $(4, 0)$, focus $(0, 0)$
   (d) vertex $(4, 0)$, directrix $x = 1$
   (e) focus $(0, 2)$, directrix $y = -4$

5. Determine the coordinates of the vertex and focus for each of the following:

   (a) $y^2 = x$
   (b) $x^2 = -y$
   (c) $(x - 3)^2 = 4y$
   (d) $(y + 1)^2 = -12x$

6. A parabolic arch is used to support a bridge. The arch is 80m wide at the base and the height of the vertex is 20m. How high is the arch at a point 20 m in from either end of the base?
7. The cross-section of a parabolic reflector is as shown. The bulb is located at the focus, F.

Find the diameter AB of the opening.

8. The latus rectum for a conic section is defined as the chord through the focus perpendicular to the axis.

Find the length of the latus rectum for each of the following

(a) \( x^2 = 4y \)
(b) \( y^2 = 12x \)
(c) \( x^2 = -2y \)

9. 2012 Hypatia Question 3
   For the question and solution go to cemc.uwaterloo.ca/contests/past_contests.html

10. 2015 CTMC- Individual Problems Question 9
    For the question and solution go to cemc.uwaterloo.ca/contests/past_contests.html