



## Grade 6 Math Circles

October 11 & 12, 2016

### *Exponents and Prime Numbers*

#### Warm Up

A **composite number** is a whole number that can be evenly divided by numbers other than 1 or itself. A **prime number** is a whole number that can be divided only by 1 and itself. (**Note:** 1 is a special case, it is neither a prime or composite number!)

1. Solve the following:

a)  $4 + 4 + 4 + 4 + 4$

d)  $7 \times 11$

b)  $8 + 8 + 8 + 8$

e)  $2 \times 2 \times 2 \times 2 \times 2 \times 2$

c)  $3 \times 6$

f)  $6 \times 6 \times 6$

2. Identify the following numbers as either composite or prime:

a) 2

b) 9

c) 24

d) 11

e) 51

f) 37

# Exponents

Exponents? What are those?

Think back to multiplication. What is multiplication again? Let's do an example.

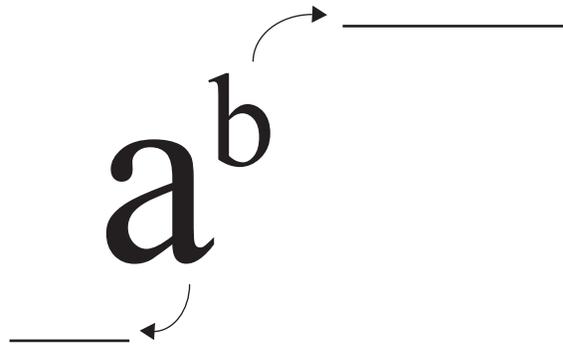
$$3 + 3 + 3 + 3 + 3 = \underline{\hspace{2cm}} \Leftrightarrow 5 \times 3 = \underline{\hspace{2cm}}$$

“Add 3 to itself 5 times” is equal to saying “5 times 3”. Multiplication is the *repeated addition* of numbers. Similarly, **exponentiation** is the \_\_\_\_\_ of numbers.

$$2 \times 2 \times 2 \times 2 \times 2$$

How do you write the above example using exponentiation? We can use something called **exponents!**

Suppose **a** and **b** can be any whole number in general. In exponentiation, the number, **a**, that is repeatedly being multiplied is called the **base**. The number of times, **b**, the base is being multiplied is called the **exponent** (can also be called the **power**). How do us mathies read this? If we see an exponent, we say, “**a** to the exponent **b**” or “**a** to the power of **b**”.



Now, is there an easier way to write  $2 \times 2 \times 2 \times 2 \times 2$  ?

Yes! We see that 2 is multiplied to itself 5 time so we write this as \_\_\_\_\_

Mathematicians have unique names for exponent 2 and exponent 3.

$a^2$

When we see “**a** to the exponent 2”, we say “**a** squared”. The product of this exponent is called a \_\_\_\_\_.

$a^3$

When we see “**a** to the exponent 3”, we say “**a** cubed”. The product of this exponent is called a \_\_\_\_\_.

**Examples** Rewrite each of the following using exponents and determine whether it is a perfect square or a perfect cube.

a)  $3 \times 3 \times 3$

b)  $7 \times 7$

## The Sieve of Eratosthenes

**The Sieve of Eratosthenes** was created some time in 276-194 BC by the Greek mathematician Eratosthenes of Cyrene. Similar to a sieve that drains water from pasta, the Sieve of Eratosthenes drains composite numbers out and leaves behind the prime numbers. It can be used to find all prime numbers up to any number we want. For this lesson, we will use it to find all prime numbers up to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Directions:**

1. Cross out 1 (since it is not prime)
2. Circle 2 and cross out every multiple of 2
3. Circle 3 and cross out every multiple of 3
4. Circle 5 and cross out every multiple of 5
5. Circle 7 and cross out every multiple of 7
6. Repeat this process until all numbers have been circled or crossed out.

What are all the prime numbers up to 100?

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## Prime Factorization

**Factors** are whole numbers that multiply together to produce another number. For example, we know that  $2 \times 3 = 6$  so 2 and 3 are factors of 6. Note that a number can have more than two factors. Is there another way to find all the factors of a number? Yes! We can check if a number is a factor using division.

Suppose we want to find all factors of 6.

$$6 \div 1 = 6$$

$$6 \div 4 = 1.5$$

$$6 \div 2 = 3$$

$$6 \div 5 = 1.2$$

$$6 \div 3 = 2$$

$$6 \div 6 = 1$$

We see that only 4 and 5 do **not** divide 6 evenly. All the factors of 6 are 1, 2, 3, and 6 because when 6 is divided by these numbers, the remainder is zero.

**Examples** Find all the factors of the following numbers:

a) 6

b) 12

c) 7

**Prime factorization** is breaking down a number into the product of its prime factors.

- \* Every positive number can be prime factored
- \* The prime factorization of a prime number is the number itself
- \* The prime factorization of 1 is 1
- \* Prime factorizations are unique meaning every number has only one prime factorization

**Example** The prime factorization of 6 is as follows:  $6 =$

There are two methods to find the prime factorization of a number.

**Method 1:** Division By Prime Numbers (DPN)

Suppose we want to find the prime factorization of 60. To find the prime factorization of 60, we divide by prime numbers. Start by dividing 60 by 2. When you cannot divide by 2 anymore, divide by 3. Then, divide by 5 and so on.

Can we write the prime factorization of 60 using exponents?

**Example** Using DPN, find the prime factorization of 100. Write your answer using exponents.

**Method 2:** Factor Tree (FT)

Using a factor tree, or FT, let's find the prime factorization of 120. Instead of dividing by prime numbers, we can use any pair of factors to find the prime factorization.

We know  $120 = 10 \times 12$  so we can start off our FT as follows...

120

The prime factorization of 120 is...

**Examples** Using a FT, find the prime factorization of 360.

## The Locker Problem

One hundred students are assigned lockers 1 to 100. The student assigned to locker 1 opens every locker. The student assigned to locker 2 then closes every other locker. The student assigned to locker 3 changes the status of all lockers whose numbers are multiples of 3 (If a locker that is a multiple of 3 is open, the student closes it. If it is closed, the student opens it). The student assigned to locker 4 changes the status of all lockers whose numbers are multiples of 4, and so on for all 100 lockers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

You may use the grid above to help solve the following questions:

1. Which lockers are left open?
2. Which lockers were touched exactly two times?
3. How do you know that these lockers were touched exactly two times?

## More Fun With Primes!

Remember what a palindrome is? Palindromes are words that are spelled the same forward and backwards such as racecar, kayak, or level. **Palindromic primes** are prime numbers that read the same forwards and backwards. Here is a list of some palindromic primes:

2, 3, 5, 7, 11, 101, 131, 151, 181, 191, 313, 353, ...

An **emirp** is a prime that gives you a *different* prime when its digits are reversed. For example, 13 becomes 31 when we reverse its digits and they are different numbers and they are both prime. A list of some emirp numbers are shown below:

13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 149, 157, ...

A **Mersenne prime** is a prime number that is one less than a power of two. To be more clear, suppose  $p$  can be any whole number. Mersenne primes can be written as follows:

$$2^p - 1$$

The first three Mersenne primes are:

$$\begin{array}{ccc} 3, & 7, & 31 \\ \downarrow & \downarrow & \downarrow \\ 2^2 - 1 & 2^3 - 1 & 2^5 - 1 \end{array}$$

### Fun fact!

Currently, the largest known Mersenne prime is  $2^{74207281} - 1$ . It is 22 338 618 digits long!

# Problem Set

1. Find the factors of the following numbers:
  - a) 56
  - b) 99
  - c) 108
  - d) 256
2. Using DPN, find the prime factorization of the following numbers:
  - a) 84
  - b) 152
  - c) 312
  - d) 1485
3. Using FT, find the prime factorization of the following numbers:
  - a) 144
  - b) 572
  - c) 1890
  - d) 6006
4. Why is 1 **not** a prime number?
5. In Mushroom Kingdom, Princess Peach is hosting a dinner party for everyone and she needs your help! Princess Peach needs to place 72 guests into equal tables. List all the possible tables that Princess Peach can sort the guests onto such that no guest is sitting by themselves.
6. Find the prime factorization of 132 by drawing two different FTs (i.e. Start with different pairs of factors). What do you notice about the prime factorization of both trees?
7. What is the next Mersenne prime after 7 ( $= 2^3 - 1$ )?
8. The volume of a box is  $1275 \text{ cm}^3$ . What are the possible dimensions of the box?  
(The volume of a rectangular box is  $length \times width \times height$ )

9. Find two numbers that have a product of 81 and a sum of 30.
10. The product of three different whole numbers is 162. What is the maximum possible sum of these three numbers?
11. What is the smallest prime number greater than 200?
12. Are the following numbers palindromic primes?
- a) 313
  - b) 752
- \*13. 761 is a prime number. Is 761 an emirp?
- \*\*14. The Locker Problem (continued)
- Recall:** One hundred students are assigned to 100 lockers.
- The student assigned to locker 1 opens every locker
  - The student assigned to locker 2 closes every other locker
  - The student assigned to locker 3 changes the status of all lockers that are a multiple of 3, so on and so on...
- a) Which lockers are left open?
  - b) What do you notice about these locker numbers that were left open?
  - c) Why were these specific lockers left open?
- \*\*15. Two numbers are said to be **co-prime** if they do not have any prime factors in common. (For example, 16 and 21 are co-prime). List all possible numbers that are co-prime with the following numbers that are less than 50:
- a) 60
  - b) 156
  - c) 390

\*\*\*16. Gru gives Margo a riddle about his minions: Kevin, Bob, and Stuart. He tells her that the product of the minions' ages is 72 and the sum of their ages is the day of the month on which her birthday falls. After some thinking, Margo tells Gru that he has not given her enough information. Gru tells her that the youngest minion really loves bananas. How old are the three minions?