Intermediate Math Circles  
Wednesday 05 October 2016  
Problem Set 1

1. In the diagram, $AB$ is parallel to $CD$. Determine the values of $x$ and $y$.

2. Triangle $ABC$ has a right angle at $B$. $AC$ is extended to $D$ so that $CD = CB$. The bisector of angle $A$ meets $BD$ at $E$. Prove that $\angle AEB = 45^\circ$.

3. In the diagram, $AB$ is parallel to $DC$ and $AB = BD = BC$. If $\angle A = 52^\circ$, determine the measure of $\angle DBC$.

4. The diagram shows three squares of the same size. What is the value of $x$?
5. The diagram shows a rhombus $FGHI$ and an isosceles triangle $FGJ$ in which $GF = GJ$. Angle $FJI$ equals $111^\circ$. What is the measure of angle $JFI$?

6. $ABCD$ is a square. The point $E$ is outside the square so that $CDE$ is an equilateral triangle. Determine the measure of angle $BED$.

7. The diagram shows two isosceles triangles in which the four angles marked $x$ are equal. The two angles marked $y$ are also equal. Find an equation relating $x$ and $y$.

8. In the diagram, $QSR$ is a straight line. $\angle QPS = 12^\circ$ and $PQ = PS = RS$. What is the measure of $\angle QPR$?
9. The diagram shows a regular nonagon with two sides extended to meet at point $X$. What is the size of the acute angle at $X$?

10. The three angle bisectors of triangle $LMN$ meet at a point $O$ as shown. Angle $LNM$ is $68^\circ$. What is the size of angle $LOM$?

11. In the figure shown, $AB = AF$ and $ABC$, $AFD$, $BFE$, and $CDE$ are all straight lines. Determine an equation relating $x$, $y$ and $z$.

12. The angles of a nonagon are nine consecutive numbers. What are these numbers?
13. What is the measure of the angle formed by the hands of a clock at 9:10?

14. Determine the sum of the angles $A$, $B$, $C$, $D$, and $E$ in the five-pointed star shown.

15. In $\triangle PQR$, $PQ = PR$. $PQ$ is extended to $S$ so that $QS = QR$. Prove that $\angle PRS = 3(\angle QSR)$.

16. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram, $TREND$ is a regular pentagon, $PEA$ is an equilateral triangle, and $OPEN$ is a square. Determine the size of $\angle EAR$. 
17. A beam of light shines from point $S$, reflects off a reflector at point $P$, and reaches point $T$ so that $PT$ is perpendicular to $RS$. What is the value of $x$?

18. In the diagram, let $M$ be the point of intersection of the three altitudes of triangle $ABC$. If $AB = CM$, then what is $\angle BCA$ in degrees?

19. In the diagram, $PW$ is parallel to $QX$, $S$ and $T$ lie on $QX$, and $U$ and $V$ are the points of intersection of $PW$ with $SR$ and $TR$, respectively. If $\angle SUV = 120^\circ$ and $\angle VTX = 112^\circ$, what is the measure of $\angle URV$?
20. Three regular polygons meet at a point and do not overlap. One has 3 sides and one has 42 sides. How many sides does the third polygon have? Can you find other sets of three polygons that have this property?

Answers

1. \( x = 10^\circ, y = 150^\circ \)  
2. \( \angle DBC = 28^\circ \)  
3. \( \angle JFI = 27^\circ \)  
4. \( x = 120^\circ \)  
5. \( \angle BED = 45^\circ \)  
6. \( \angle QPR = 54^\circ \)  
7. \( y = 2x \)  
8. \( \angle LOM = 124^\circ \)  
9. \( 60^\circ \)  
10. \( x - y + 2z = 180^\circ \)  
11. \( 136^\circ \) to \( 144^\circ \)  
12. \( \angle EAR = 39^\circ \)  
13. \( x = 32^\circ \)  
14. \( \angle BCA = 45^\circ \)  
15. \( \angle URV = 52^\circ \)  
16. \( 7 \) sides