Intermediate Math Circles  
**Wednesday November 23 2016**  
Problem Set 8

1. The point of this first exercise is to get warmed up and recall what we learned last time.

   (a) Find the remainder when 3434 is divided by 17.
   
   (b) What is the tens digit of \((99)^{881}\)?
   
   (c) January 1, 2017 is on a Sunday. What day of the week will February 14, 2018 be on?

2. Calculate the remainder when \((17)^{125}\) is divided by 5. (Hint: Use Fermat’s Little Theorem)

3. True or False: Every number of the form \(a^2 - a\) is even.

4. What is \(15002^{34}\) congruent to modulo 3? (i.e. 0, 1, or 2?)

5. Show that the check digit *always* detects an error made when making exactly one typo in a UPC.

6. Which of the following UPC’s have correct check digits?

7. What is the check digit of the UPC \((0, 3, 0, 9, 5, 5, 1, 6, 9, 8, 2, \ast)\)? What do you notice about this answer when compared to the above question?

8. Determine when the check digit of a UPC does *not* detect an error made by switching two adjacent numbers.

9. In Florida, the fourth and fifth digits from the end of a driver’s license number give the year of birth. The last three digits for a male with birth month \(m\) and birth date \(b\) are represented by \(40(m - 1) + b\). For females the digits are \(40(m - 1) + b + 500\). Determine the dates of birth of people who have last five digits \(42218\) and \(53953\).

10. For driver’s license numbers issued in New York prior to September of 1992, the three digits preceding the last two of the number of a male with birth month \(m\) and birthdate \(b\) are represented by \(63m + 2b\). For females the digits are \(63m + 2b + 1\). Determine the dates of birth and genders which correspond to the numbers \(248\) and \(601\).

11. The state of Utah appends a ninth digit \(a_9\) to an eight-digit driver’s license number \(a_1a_2\ldots a_8\) so that \((a_1, a_2, \ldots, a_9) \cdot (9, 8, 7, 6, 5, 4, 3, 2, 1) \equiv 0 \pmod{10}\). If you know that the license number \(149105267\) has exactly one digit incorrect, explain why the error cannot be in position 2,4,6, or 8.