



Complex Numbers and e

Problem Set

1. Statistics show that the world population increased at a continuous rate of 1.9% since WWII. In 1975, the United Nations announced that the world population was 4 billions people.
 - (a) Using this model, what was the population in 2000?
 - (b) According to this model when was the population supposed to reach 7 billions people?
 - (c) The world population actually reached 7 billions in 2010. At which rate did the population increased from 1975 to 2010?
2. A spelunker finds a new kind of crystal that weights 300 kg in a cave that grows at a continuous rate so that it can generate a crystal of identical mass in 24h. What would be the total mass of crystal produced in 30 days?
3. A very generous bank offers an interest rate of 5% per year, but the client gets to choose the compounding frequency. If a client invests 120\$ at this bank, what is the maximum value of that investment in 10 years?
4. A chemist creates 10 kg of substance who decay continuously at a rate of 100% per year.
 - (a) How much of that substance would there be left in 3 years ?
 - (b) What is this substance's half-life ? (the half-life of substance is defined as the time it takes for the substance quantity to be reduced by half it's original quantity).

5. Newton's law of cooling states that the rate of change (which is continuous) of an object's temperature is proportional to the temperature of its surroundings that can be modeled by the following function:

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

where:

- $T(t)$: temperature of the object at a certain time t .
- T_s : temperature of the surroundings.
- T_0 : initial temperature of the object.
- k : rate of change of the object's temperature.

Mr Wallace's body was found in his house at 8:30 pm. When the police got there, Mr Wallace's body's temperature was 32 degrees Celsius and the cave where he was found was 10 degrees Celsius. 90 minutes later, the temperature of the body was 28 degrees Celsius. Mrs Wallace was seen leaving the house at 6 pm. Could she have done it?

6. Find all fourth roots of 1.
7. Express $\left(e^{\frac{i\pi}{12}}\right)^9$ in standard form.
8. Express \sqrt{i} in standard form.
9. How many possible values are there for $(1+i)^{\frac{2}{3}}$?
10. Show that i^i is a real number.
11. Express $(1+i)^{(1+i)}$ in polar form.
12. Show that $i^{-i} = \sqrt{e^\pi}$ is actually the same equation as $e^{i\pi} + 1 = 0$