**Jmc 1977 #22**

**Solution.**
The diagram represents a cross section of the tank. The distance AB is twice 
\[ BD = 2\sqrt{2^2 - 1^2} = 2\sqrt{3}. \]
Surface area = area of a rectangle of dimensions 16 and \(2\sqrt{3}\), that is, \(32\sqrt{3}\).

The answer is (A).

---

**Jmc 1981 #24**

**Solution**
In 90! there are 45 factors which are multiples of 2. Of these 22 are multiples of 4, 11 are multiples of 8, 5 are multiples of 16, 2 are multiples of 32, and 1 is a multiple of 64. Then the exponent of the highest power of 2 is 
\[45 + 22 + 11 + 5 + 2 + 1 = 86. \] (A)

---

**Jmc 1976 #18**

**Solution 1.**
The line has equation 
\[ \frac{x}{a} + \frac{y}{b} = 1. \]
Since (2, 1) lies on the line, 
\[ \frac{2}{a} + \frac{1}{b} = 1, \]
that is, 
\[ 2b + a = ab, \]
\[ 2b = ab - a, \]
\[ 2b = a(b - 1). \]
The answer is A.

---

**Solution 2.**
Slope \(AC = \frac{1 - 0}{2 - a}\); slope \(BC = \frac{1 - b}{2 - 0}\).
Equating slopes, we obtain 
\[ \frac{1}{2 - a} = \frac{1 - b}{2}, \]
\[ 2 = 2a - 2b + ab, \]
\[ 2b = a(b - 1). \]
The answer is A.
Solution 1.

Let DC = x, and use the theorem on intersecting chords.

BC = CA = CG = EF = 9 + x

= radius of larger circle.

From the small circle,

\[ DC \cdot CA = EC \cdot CH. \]

\[ x(9 + x) = (9 + x - 5)^2 \]

\[ 9x + x^2 = (4 + x)^2 = x^2 + 8x + 16 \]

By transposition, \( x = 16 \).

Hence the required diameter

\[ = DC + CA \]

\[ = 16 + (9 + 16) = 41. \]

The answer is (C).

Solution 2.

Let O be the centre of the smaller circle. Designate the radii of the larger and smaller circles by R and r respectively.

\[ \therefore 2R = 2r + 9 \]

Hence \( R = r + \frac{9}{2} \).

Now CE = R - 5

\[ = r + \frac{9}{2} - 5 \]

\[ = r - \frac{1}{2} \]

and OC = R - r

\[ = r + \frac{9}{2} - r \]

\[ = \frac{9}{2} \]

In the right triangle ECO,

\[ EO^2 = CE^2 + OC^2 \]

\[ r^2 = (r - \frac{1}{2})^2 + (\frac{9}{2})^2 \]

\[ = r^2 - r + \frac{1}{4} + \frac{81}{4} \]

\[ \therefore r = \frac{41}{2} \]

Hence the required diameter is \( 2r = 41 \).

The answer is (C).
Let the $x$ and $y$ intercepts of the line be $a$ and $b$ respectively.

Then slope $AP = \text{slope } AB$

\[
\frac{6}{-2 - a} = \frac{b}{-a}
\]

\[b = \frac{6a}{a + 2}
\]

Since the area of $\triangle AOB$ is 25,

then \[\frac{1}{2} \cdot a \cdot b = 25.
\]

\[a \left(\frac{6a}{a + 2}\right) = 50
\]

\[6a^2 + 50a + 100
\]

\[3a^2 - 25a - 5 = 0
\]

\[(3a + 5)(a - 10) = 0
\]

\[a = \frac{5}{3} \text{ or } 10.
\]

The correct answer is (C).

---

**JMC 1975 #26**

**Solution 1.**

\[
\frac{x - 18}{x^2 - x - 6} = \frac{P}{x + 2} + \frac{Q}{x - 3}
\]

\[
= \frac{P(x - 3) + Q(x + 2)}{x^2 - x - 6}
\]

\[
= \frac{(P + Q)x + (2Q - 3P)}{x^2 - x - 6}
\]

Since this is an identity,

\[x - 18 = (P + Q)x + (2Q - 3P).
\]

Hence \[2Q - 3P = -18,
\]

\[Q + P = 1.
\]

Solving, we get \[P = 4, \ Q = -3.
\]

Hence \[P - Q = 4 - (-3) = 7.
\]

**Solution 2.** As in Solution 1,

\[x - 18 = P(x - 3) + Q(x + 2)
\]

Since this identity holds for all $x$, it holds for $x = 3$.

Thus \[-15 = 5Q, \ Q = -3.
\]

Also, the identity holds for $x = -2$;

hence \[-20 = -5P, \ P = 4.
\]

As before, \[P - Q = 7.
\]

---

**PASCAL 1982 #25**

List the integers as follows:

\[
\begin{align*}
000000 & \\
000001 & \\
000002 & \\
\vdots & \\
999998 & \\
999999 & \\
\end{align*}
\]

There are 6000000 digits in this list and the digits 0, 1, 2, ..., 9 each appear an equal number of times. So
There are 5 \( (4) (3) = 60 \) possible numbers.
By symmetry, each digit must appear \( \frac{60}{5} = 12 \) times
in each of the first, second, and third positions.
So the digits in each position add to 12 \((2 + 3 + 4 + 5 + 6) = 240\)
Units digits give 240.
Tens digits give 2400.
Thousands digits give 24000.
Total sum is 26,640.
The answer is D.

Note: The 60 numbers are
234 245 345 456
235 246 346
236 256 356
and all rearrangements of these (234 gives itself,
243, 342, 324, 432, and 423).

**JMC 1976 #23**

**Solution 1.**

Since \( 29^2 = 21^2 + 20^2 \),
we find that \( \triangle \) is a right angle.
Let \( BF = BD = x \),
\( DC = CE = r \), \( FA = EA = y \). (tangents from external points are equal)
Then \( r + x = 20 \), \( r + y = 21 \), \( x + y = 29 \).
Thus \( 2r + x + y = 41 \), \( 2r = 12 \), \( r = 6 \) (and \( 2r = 12 \)).
The answer is A.

**Solution 2.**

\[ \triangle BCA = \triangle BOC + \triangle OCA + \triangle OAB. \]
\[ \frac{1}{2} (20) (21) = \frac{1}{2} r (20) + \frac{1}{2} r (21) + \frac{1}{2} r (29). \]
\[ 420 = r (70), \ r = 6 \text{ (and } 2r = 12). \]
The answer is A.
(Note that one could get \( \triangle BCA \) by Heron's formula without even noting that it is right-angled.)
Let \( 1 + k = 3a \), \( 1 + 2k = 5b \), and \( 1 + 8k = 7c \) where \( a, b, c \) are integers.

\[ a = \frac{k+1}{3} \quad \Rightarrow \quad k = 2, 5, 8, 11, \ldots, 59, 62, 65, \ldots \]

\[ b = \frac{2k+1}{5} \quad \Rightarrow \quad k = 2, 7, 12, 17, \ldots, 57, 62, 67, \ldots \]

\[ c = \frac{8k+1}{7} \quad \Rightarrow \quad k = 6, 13, 20, 27, \ldots, 55, 62, 69, \ldots \]

The smallest value of \( k \) satisfying all three conditions is 62.

\[ \text{(D)} \]

\[ \text{JMC 1979 \#27} \]

\[ \frac{1}{(2)(3)} = \frac{1}{2} - \frac{1}{3} \]

\[ \frac{1}{(3)(4)} = \frac{1}{3} - \frac{1}{4} \]

\[ \frac{1}{(61)(62)} = \frac{1}{61} - \frac{1}{62} \]

Add to obtain \( \frac{1}{2} - \frac{1}{62} = \frac{31}{62} - \frac{1}{62} = \frac{30}{62} = \frac{15}{31} \).

Thus \( a = 15 \), \( b = 31 \), \( a + b = 46 \).

The answer is \( \text{(D)} \).