Intermediate Math Circles
Analytic Geometry I

Problems

1. Three points are collinear if they all lie on a straight line. Show that $P(-12, 1)$, $Q(-4, -3)$ and $R(6, -8)$ are collinear.

   (a) Use a slope argument to show collinearity.
   (b) Use a distance argument to show collinearity.

2. The point $A(-2, y)$ is on a line that passes through the points $T(0, -2)$ and $W(4, 0)$. Determine the value of $y$.

3. $\triangle ABC$ has vertex $A$ on the $x$-axis at $-2$ and vertex $C$ on the $x$-axis at $8$. The third vertex $B$ is on the $y$-axis at $b$ such that $\angle ABC = 90^\circ$. Determine all possible values of $b$.

4. A point $W$ is located on the $x$-axis so that it is 13 units from the point $R(7, 5)$. Find the coordinates of point $W$.

5. The points $A$ and $B$ are located in the first quadrant, equidistant from the origin, $O$. If the slope of $OA$ is 7 and the slope of $OB$ is 1, determine the slope of $AB$.

6. The vertices of $\triangle ABC$ are $A(-2, -11)$, $B(10, 5)$ and $C(12, 3)$.

   (a) Determine the midpoint $M$ of line segment $AB$.
   (b) Show that $AM = MB = MC$. This will prove that $M$ is the centre of a circle containing points $A$, $B$ and $C$ on the circumference.
   (c) Show that $\angle ACB = 90^\circ$.

7. The line segment $AB$, where $A$ is $(2, -4)$ and $B$ is $(10, 8)$, is divided at $Q$ in the ratio $3 : 5$. Find the coordinates of $Q$. 