Intermediate Math Circles
February 04, 2015
Contest Preparation I

WARM-UP: Hockey is Back!

Four teams A, B, C, and D competed against each other. Unlike the NHL, games in this league can end in a tie. The following table summarizes the results of the games:

<table>
<thead>
<tr>
<th>Team</th>
<th>Wins</th>
<th>Ties</th>
<th>Losses</th>
<th>Goals For</th>
<th>Goals Against</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

In the final game, B played C and the score was 1-1.

a) Which team won in each of the six matches?

b) What were the scores in the matches that B played?

c) What were the scores in the rest of the matches?
General Contest Information

• Name of the contest you would write ____________________

• Date of Contest ____________________

• Registration Deadline ____________________

• Contest Details

• The key to Success ____________________

• Available resources to help you prepare


Go to [http://www.cemc.uwaterloo.ca/contests/past_contests.html](http://www.cemc.uwaterloo.ca/contests/past_contests.html) for old contests and solutions available for over 15 years.

Math Circles for contest preparation sessions / workshops.
Eliminating Answers
From PCF EWorkshop Practice Cayley #2 Question 1
If $x = -2$ and $y = -5$ then $(x - y)(x + y)$ equals
(A) 40    (B) 21    (C) 0    (D) -21    (E) -49

(Pascal #3)
If $a = 4$, $b = 5$, and $c = 9$, then the value of $(a + b - c) + (a - b + c)$ is
(A) 8    (B) 0    (C) -10    (D) 10    (E) 16

(Pascal #4)
The number of odd integers between $\frac{19}{4}$ and $\frac{43}{2}$ is
(A) 8    (B) 9    (C) 10    (D) 16    (E) 17

(Pascal #6)
The ratio of the number of red marbles to the number of green marbles in a container is $1 : 3$. If there are 48 marbles altogether, then the number of green marbles is
(A) 12    (B) 16    (C) 36    (D) 32    (E) 24
(Pascal #5)
In a magic square, the sum of the entries in each row, in each column, and in each diagonal are all equal.

<table>
<thead>
<tr>
<th>15</th>
<th>1</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the magic square shown, the value of $N$ is
(A) 5  (B) 13  (C) 7  (D) 16  (E) 3

(Pascal #19)
During a high school drama rehearsal, 15 girls left. Twice as many boys as girls remained, Later, 45 boys departed, leaving five times as many girls as boys. Before anyone left the rehearsal, there were
(A) 10 more girls than boys  (B) 20 more boys than girls
(C) 25 more boys than girls  (D) 10 more boys than girls
(E) 20 more girls than boys

(Pascal #16)
The number of integers between 2 and 50 that can be written in the form $x^y$, where $x$ and $y$ are positive integers and $y \neq 1$, is
(A) 8  (B) 7  (C) 10  (D) 6  (E) 9
(Pascal #14)
The eight digits 6, 5, 5, 4, 4, 3, 2, 1 are used to form two three-digit numbers and one two-digit number. The largest possible sum of these numbers is

(A) 1119  (B) 1713  (C) 1218  (D) 30  (E) 1236

(Pascal #18)
Each of the numbers from 1 to 9 is placed, one per circle, into the pattern shown. The sums along each of the four sides are equal. The number of possible entries for the middle circle is

(A) 2  (B) 9  (C) 1  (D) 5  (E) 3

(Pascal #7)
The diagram shows a square $PQRS$ and two equilateral triangles $RSU$ and $PST$. $PQ$ has length 1. The length of $TU$ is

(A) $\sqrt{2}$  (B) $\frac{\sqrt{3}}{2}$  (C) $\sqrt{3}$  (D) $\sqrt{5} - 1$  (E) $\sqrt{6} - 1$
(Pascal #21)  
In the “Big 15” Lottery, each of the numbers from 1000 to 9999 inclusive is printed, one to a ticket. A number is considered a winner if its hundreds digit is 8, its tens digit is 6, and it is divisible by 15. The number of winning tickets is

(A) 3  (B) 9  (C) 2  (D) 15  (E) 6

(Pascal #23)  
In the sequence 6, 14, 8, −6, ⋯, every term after the second is the difference of the preceding two terms in reverse order. For example, 8 = 14 − 6 and −6 = 8 − 14. The sum of the first 2000 terms of the sequence is

(A) 28  (B) 8  (C) 22  (D) 0  (E) 20

(Pascal #24)  
On planet Binad, a Bank Cash machine uses eight ON-OFF switches numbered 1 to 8 to allow deposits to and withdrawals from an account. All the switches start out OFF. When switch $n$ is flipped ON, the balance of the account changes by $(-2)^n$ dollars. For example, if switches 4 and 7 are flipped ON and the other switches are left OFF, a withdrawal of 112 dollars is made. The number of switches that must be turned ON to deposit 114 dollars is

(A) 4  (B) 6  (C) 5  (D) 3  (E) 7

Questions: send an email to troy.vasiga@uwaterloo.ca