CIMC Sample Contest

Part A

1. Determine the value of $\frac{\sqrt{25} - 16}{\sqrt{25} - \sqrt{16}}$.
   \{2008 Cayley #2\}

2. In the diagram, $PT$ and $QS$ are straight lines intersecting at $R$ such that $QP = QR$ and $RS = RT$.
   Determine the value of $x$.
   \{2008 Cayley #8\}

3. If $x + y + z = 25$, $x + y = 19$ and $y + z = 18$, determine the value of $y$.
   \{1998 Cayley #11\}

4. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of $x$?
   \{2010 Cayley #16\}

5. What is the largest positive integer $n$ that satisfies $n^{200} < 3^{500}$?
   \{2010 Cayley #20\}

6. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?
   \{2010 Cayley #24\}
Part B

1. (a) Determine the average of the integers 71, 72, 73, 74, 75.
(b) Suppose that $n, n + 1, n + 2, n + 3, n + 4$ are five consecutive integers.
   (i) Determine a simplified expression for the sum of these five consecutive integers.
   (ii) If the average of these five consecutive integers is an odd integer, explain why $n$ must be an odd integer.
(c) Six consecutive integers can be represented by $n, n + 1, n + 2, n + 3, n + 4, n + 5$, where $n$ is an integer. Explain why the average of six consecutive integers is never an integer.

\{2010 Fryer #2\}

2. (a) Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.

(b) Point $C(0, p)$ lies on the y-axis between $Q(0, 12)$ and $O(0, 0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of $p$.

(c) Determine an expression for the area of $\triangle QCA$ in terms of $p$.

(d) If the area of $\triangle ABC$ is 27, determine the value of $p$.

\{2010 Galois #2\}
Part B (continued)

3. If \( m \) is a positive integer, the symbol \( m! \) is used to represent the product of the integers from 1 to \( m \). That is, \( m! = m(m-1)(m-2)\ldots(3)(2)(1) \). For example, \( 5! = 5(4)(3)(2)(1) \) or \( 5! = 120 \).

Some positive integers can be written in the form

\[
    n = a(1!) + b(2!) + c(3!) + d(4!) + e(5!).
\]

In addition, each of the following conditions is satisfied:

- \( a, b, c, d, \) and \( e \) are integers
- \( 0 \leq a \leq 1 \)
- \( 0 \leq b \leq 2 \)
- \( 0 \leq c \leq 3 \)
- \( 0 \leq d \leq 4 \)
- \( 0 \leq e \leq 5 \).

(a) Determine the largest positive value of \( N \) that can be written in this form.

(b) Write \( n = 653 \) in this form.

(c) Prove that all integers \( n \), where \( 0 \leq n \leq N \), can be written in this form.

(d) Determine the sum of all integers \( n \) that can be written in this form with \( c = 0 \).

{2009 Galois #4}