SOLUTION SET 1

Here are the solutions to exercises 2 to 5 from the first problem set. As explained before, Exercise 1 was meant to be a refresher on some basic concepts, and Exercises 6 and 7 were not reached. They will appear again on the second handout.

Solution 2.

(1) $2, 1, 7, 10, 31, 61, 154, 337, 799, 1810 \ldots$

(2) $3, 2, 1, 0, -1, -2, -3, -4, -5, -6, \ldots$

(3) $1, 1, 1, 1, 1, 1, 1, 1, 1, 1, \ldots$

(4) $1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, \ldots$

(5) $1, 2, 4, 8, 16, 32, 64, 128, 256, 512, \ldots$

(6) $1, 1, 5, 1, 29, -23, 197, -335, 1517, -3527, \ldots$

(7) $3, 2, -4, -8, 0, 16, 16, -16, -48, -16, \ldots$

(8) $4, 7, -11, 4, 7, -11, 4, 7, -11, 4, \ldots$

(9) $1, 1, -1, -5, -7, 1, 23, 43, 17, -95,$

(10) $1, 1+\sqrt{2}, 3+2\sqrt{2}, 7+5\sqrt{2}, 17+12\sqrt{2}, 41+29\sqrt{2}, 99+70\sqrt{2}, 239+169\sqrt{2} \ldots$. As for the bouns,

$n = 1 \;\; (1+\sqrt{2})^1 = 1+\sqrt{2}$

$n = 2 \;\; (1+\sqrt{2})^2 = 3+2\sqrt{2}$

$n = 3 \;\; (1+\sqrt{2})^3 = (1+\sqrt{2})^2(1+\sqrt{2}) = (3+2\sqrt{2})(1+\sqrt{2}) = 7+5\sqrt{2}$

$n = 4 \;\; (1+\sqrt{2})^4 = (1+\sqrt{2})^3(1+\sqrt{2}) = (7+5\sqrt{2})(1+\sqrt{2}) = 17+12\sqrt{2}$

Look familiar?

Solution 3.

(1) $a_n = a_{n-1} + a_{n-2}$

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	2	3	5	8	13	21	34	55	89
2	2	4	6	10	16	26	42	68	110
2	4	6	10	16	26	42	84	110	188
3	4	7	11	18	29	47	76	123	199
−1	0	−1	−1	−2	−3	−5	−8	−13	−21
4	6	10	16	26	42	68	110	178	288

(2) $a_n = 3a_{n-1} - 2a_{n-2}$

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	1	1	1	1	1	1	1	1	1
1	2	4	8	16	32	64	128	256	512
-1	-2	-4	-8	-16	-32	-64	-128	-256	-512
2	3	5	9	17	33	65	129	257	513
-1	0	2	6	14	30	62	126	254	510

Solution 4. We have already computed the sequence when $a_0 = a_1 = 1$ and $a_0 = 1, a_1 = 2$. They are $1, 1, 1, 1, \ldots$ and $1, 2, 4, 8, 16, \ldots$ (the powers of 2). Take these sequences as a_n and b_n, and using the hint we have that $c_n = 5a_n - 2b_n$. The first few terms:

$$\begin{aligned}
c_0 &= 5a_0 - 2b_0 \\
&= 5 \cdot 1 - 2 \cdot 1 \\
&= 3 \\
c_1 &= 5a_1 - 2b_1 \\
&= 5 \cdot 1 - 2 \cdot 2 \\
&= 1 \\
c_2 &= 5a_2 - 2b_2 \\
&= 5 \cdot 1 - 2 \cdot 4 \\
&= -3 \\
c_3 &= 5a_3 - 2b_3 \\
&= 5 \cdot 1 - 2 \cdot 8 \\
&= -11 \\
c_4 &= 5a_4 - 2b_4 \\
&= 5 \cdot 1 - 2 \cdot 16 \\
&= -27
\end{aligned}$$

Solution 5. Notice that $a_n = 1$ for all n, and $b_n = 2^n$ for all n. Using that $c_n = 5a_n - 2b_n$, we have that $c_n = 5 \cdot 1 - 2 \cdot 2^n = 5 - 2^{n+1}$. Try this on your calculator, if you haven't already.