

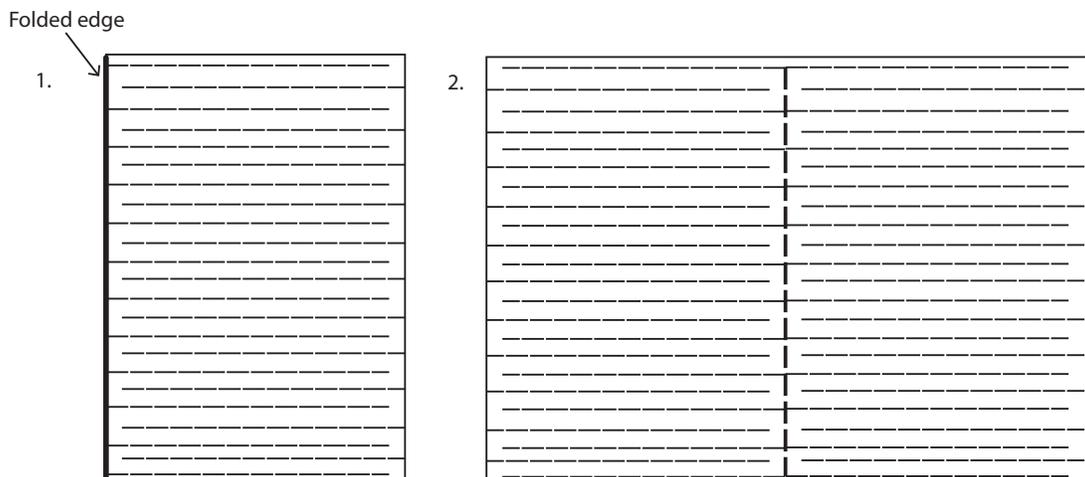


Grade 6 Math Circles
February 25/26, 2014
Unique Geometry Solutions

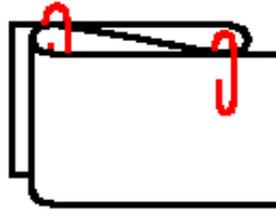
1. **Mobius' Crosses:** To begin, take two strips of paper and tape them together at a right angle so they make a "+". Be sure to securely tape all of the edges. You will need three of these crosses altogether.
 - (a) With your first cross, take two opposite arms and tape them together in an untwisted loop. Take the other two arms and tape them together in an untwisted loop. Your cross should now look like a figure-8 that is half twisted in the middle. What do you think will happen when you cut both loops down their middles? Try it to test your hypothesis. *You should end up with a square.*
 - (b) Using your second cross, again tape opposite ends into loops, but this time make one loop and one Mobius band. What do you think will happen when we cut these loops down their middles? Test your hypothesis. *You should end up with a square that may need to be untangled*
 - (c) With your third cross, tape both pairs of opposite ends into Mobius bands. Make a hypothesis as to what will happen when they are cut in half and test your hypothesis. *You should get two loops that may be linked together or separated.*

2. **Double Mobius Bands:** Hold two strips of paper together in your hand (one underneath the other), and bring the ends together with a half-twist (like you are making a Mobius band). Get a friend to help you tape the top two strips together and the bottom two strips together. You should now have two Mobius bands nestled snugly side by side.
 - (a) Take a pencil and put it between the two loops. Move the pencil around the loop one time until it returns to the place where you began. Is your pencil facing the same or opposite direction than it was before? Can you explain this? What happens when you move your pencil around the loop a second time? *Your pencil should be facing the opposite direction after one time through, and should return to its original direction after the second time.*

- (b) Make a hypothesis as to what will happen when we pull the two bands apart. Test your hypothesis.
- Making sure that you have taped the top ends together and the bottom end together (without joining all four ends) you should get one long twisted loop.
- Take a small piece of paper (at most one quarter of a full sheet) and in the middle trace around the rim of a penny. Then carefully and accurately cut a circular hole in your paper along that outline. Your challenge now is to pass a quarter through this hole without tearing the paper. Fold the paper in half so that half of the hole is on either side. Drop the coin into the hole. Holding both ends (one of your hands should be on either side of the hole), bring your hands together upwards. The coin should squeeze through the hole.
 - For this puzzle, use half of a piece of paper. Your challenge is to cut a hole in this paper so large that a person can fit through it. **Hint:** Cutting a hole does not necessarily mean we have to remove parts of the paper. Fold the paper in half. Starting on the folded side, cut almost all the way through. Then make a cut starting from the open (unfolded) side almost all the way through. Continue until you get to the end. The closer your cuts are the bigger the hole will be. One you have cut the whole paper, making sure your last cut starts on the folded side, carefully unfold the paper. Skipping the first and last strips, cut down the middle. You should end up with a big loop. You should make cuts along the dotted lines as shown in the diagram below. In the second step, you should cut along the bolded dotted line.



5. Using a strip of paper and two paperclips, fold the strip over and then back, so you get a “Z” shape, like the picture below. Now slide the paperclips on from one side, so that the paper is held in this shape.
- (a) Make a hypothesis as to how far apart the paperclips will land when you pull hard on both ends of the paper, making them fly into the air. Try it out, are you surprised by your results? [The paper clips become linked together!](#)
- (b) Try to achieve the same result using three or more paperclips, and more than two folds.



6. Try to recreate the figure below using a single sheet of paper and no tape. [Follow the diagram below, cutting on the solid lines and folding on the dotted line.](#)

