1. What is Cryptography?
2. A bit on Modular Arithmetic
3. Let’s do some encryption and decryption!
4. Let’s break a cipher!
What is Cryptography?

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- Scytale (a Transposition Cipher) dates to 5th century BC, used by Spartan military
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In the 20th century, cryptography played a significant role in many global conflicts (e.g. Enigma machine and Bletchley Park in WWII).
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Credit card, debit card and web transactions, as well as privacy concerns for the electronic storage of health, citizenship and other records, have raised the need for secure communications and secure storage dramatically.
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Alice needs to "encrypt" the message so that Eve cannot read it. Alice wants to use a simple algorithm, so that Bob can "decrypt" the transmitted message using some special key that only he has, but so that it is hard for Eve to "break the code" without knowing the key.
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Plaintext = original message
Ciphertext = encrypted message

Encryption = act of transforming plaintext into ciphertext
Encryption algorithm = method used to turn plaintext into ciphertext. Uses a key, some input into the algorithm.

Decryption = act of transforming ciphertext into plaintext
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Diagram:

- Alice
  - Message (Plaintext)
  - Encrypt
  - Ciphertext

- Bob
  - Decrypt
  - Message

- Eve
  - Communication channel
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![Diagram of encryption process](image)
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**Decryption algorithm** = method used to turn ciphertext into plaintext. Uses a **key**, some input into the algorithm.
The security of the code should depend on keeping the key a secret, not keeping the encryption algorithm a secret.

Alice should be able to code her message with a well-known method and still be reasonably confident that the message cannot be decoded by anyone other than Bob, since he is the only person with the key.
What is the remainder when 51 is divided by 7?

Answer: 2

We write $51 \equiv 2 \pmod{7}$ and say “51 is congruent to 2 modulo 7.” This means that 51 and 2 have the same remainder when divided by 7. In other words, $51 - 2 = 49$ is a multiple of 7.

In general, if $a$, $b$, and $m$ are integers, $a \equiv b \pmod{m}$, “$a$ is congruent to $b$ modulo $m$” means that $a$ and $b$ differ by a multiple of $m$, or $a - b = km$, where $k$ is some integer.
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We will be interested in the smallest integer $b \geq 0$ such that

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Example 1

Reduce

a) $52 \mod 8$

b) $41 \mod 5$

c) $84 \mod 4$

d) $-17 \mod 4$

e) $145672 \mod 13$
First Some Modular Arithmetic

$28 \equiv 4 \pmod{8}$ and $11 \equiv 3 \pmod{8}$.
What happens when we add or subtract?

Reduce $28 + 11$ modulo 8.

$28 + 11 = 39 \equiv 7 \pmod{8}$.

Also notice:

$28 \equiv 4 \pmod{8}$ and $11 \equiv 3 \pmod{8}$ and $3 + 4 = 7 \equiv 7 \pmod{8}$

Reduce $28 - 11$ modulo 8.

$28 - 11 = 17 \equiv 1 \pmod{8}$.

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Reduce (in two different ways)

a) \( 17 + 21 \text{ mod } 6 \)

b) \( 83 - 21 \text{ mod } 3 \)

c) \( 21 - 83 \text{ mod } 11 \)
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Exercise Set 1

1. Reduce 237288 modulo 5
2. Reduce 192 + 118 modulo 5
3. Reduce 192 − 118 modulo 5
4. Reduce 118 − 192 modulo 5
5. Today is a Wednesday. What day of the week will it be
   a) 100 days from now?
   b) 365 days from now?
   c) 1000 days from now?
6. Emily celebrated her 13th birthday on Wednesday, February 19th, 2014. On what day of the week was she born? (Don’t forget about the leap years in 2004, 2008 and 2012!)
Answers to Exercise Set 1:

1. \[237288 \equiv 3 \mod 5\]
2. \[192 + 118 \equiv 0 \mod 5\]
3. \[192 - 118 \equiv 4 \mod 5\]
4. \[118 - 192 \equiv 1 \mod 5\]
5. a) Friday
   b) Thursday
   c) Tuesday
6. Monday
Recall:
We have a sender, Alice, and a receiver, Bob. Alice wants to send a message $M$ to Bob, over an insecure channel, but she wants only Bob to be able to read the message. There is a good chance that at least part of the transmitted message will be intercepted by and eavesdropper, Eve.

Alice needs to “encrypt” the message so that Eve cannot read it.
The Caesar Shift Cipher

Assign the numbers 0 to 25 to the letters A to Z (so A is 0, B is 1 and so on, Z is 25). Think of the alphabet mod 26.

Pick a random number to be your key, call it $k$.

Encryption Algorithm:
Encrypt each letter individually using the formula:

$$\text{coded} = (\text{original} + k) \pmod{26}$$

Decryption Algorithm:
Decrypt each letter individually using the formula:

$$\text{original} = (\text{coded} - k) \pmod{26}$$

Jen Nelson jen.nelson@uwaterloo.ca
Intermediate Math Circles March 19, 2014 Cryptography I
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Assign the numbers 0 to 25 to the letters A to Z (so A is 0, B is 1 and so on, Z is 25).
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Pick a random number to be your key, call it $k$.

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- A is encrypted as H since \( 0 + 7 \equiv 7 \pmod{26} \)
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How does Bob decrypt the ciphertext into plaintext?
Example 3

a) Using a Caesar Shift Cipher and secret key 11, encrypt the message “I WANT COOKIES”.

b) Using a Caesar Shift Cipher and secret key 11, decrypt the message “NSPNV ESP NFAMZLCO”.

A   B   C   D   E   F   G   H   I   J   K   L   M

N   O   P   Q   R   S   T   U   V   W   X   Y   Z
Exercise Set 2

1. Encode the message “MODULAR ARITHMETIC” using a Caesar Shift cipher with secret key $k = 7$.

2. Decode the message “SLA AOLT LHA JHRL” using a Caesar Shift cipher with secret key $k = 7$.

3. Encode the message “ALL SQUARES ARE RECTANGLES” using a Caesar Shift cipher with secret key $k = 14$.

4. Decode the message “PIH BCH OZZ FSQHOBUZSG OFS GEIOFSG” using a Caesar Shift cipher with secret key $k = 14$.

5. Find a partner to work with. Think of a secret message to send to them and encode it using a Caesar Shift cipher with a secret key of your choice. Give your partner the coded message and shift number. Decode your partner’s message to you.
Answers to Exercise Set 2:

1. TVKBSHY HYPAOTLAPJ
2. LET THEM EAT CAKE
3. OZZ GEIOFSG OFS FSQHOBUZSG
4. BUT NOT ALL RECTANGLES ARE SQUARES
Encryption and Decryption are very easy with the Caesar Shift Cipher.

Sadly, this is also very easy to break. Can you see how?

Just to try all possible keys!

There are only 26 - you don't even need a computer to try this!
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How can we improve on the Caesar shift cipher?
A Random Substitution Cipher

How can we improve on the Caesar shift cipher?

Instead of shifting the alphabet, use a random permutation of the alphabet to get a Substitution Cipher.

For example:

A
B
C
D
E
F
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The table above acts as the key for this cipher.

How many possible keys are there?

$26! \approx 4 \times 10^{26}$

Breaking the cipher by trying all keys is no longer feasible, even for a computer!
How can we improve on the Caesar shift cipher?

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```
A | B | C | D | E | F | G | H | I | J | K | L | M
---|---|---|---|---|---|---|---|---|---|---|---|---
O | Y | C | P | K | G | V | W | B | Q | U | Z | J
```

```
N | O | P | Q | R | S | T | U | V | W | X | Y | Z
---|---|---|---|---|---|---|---|---|---|---|---|---
X | E | N | M | H | T | D | S | I | L | F | R | A
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Breaking the cipher by trying all keys is no longer feasible, even for a computer!
## Example 4:

a) Using the Substitution Cipher above, encrypt the message “I WANT COOKIES”.

b) Using the Substitution Cipher above, decrypt the message “BX DWK CEEUBK QOH”.

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Breaking a Substitution Cipher

The Substitution Cipher is better than the Caesar Shift Cipher, but unfortunately, it can also be easily broken. How?

A statistical analysis can be used, using known letter frequencies in the English alphabet.

Order of relatively frequency:

1. E
3. D, L
5. V, K, J, X, Q, Z
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Breaking a Substitution Cipher

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Order of relatively frequency:

1. E
3. D, L
5. V, K, J, X, Q, Z
Exercise Set 3 Using known letter frequencies in the English alphabet, try to break the code below.
The message was encrypted with a substitution cipher.

IFYYOL PYZZR AXRGVK QWBZ IQL IFWK FB ZWEV PXB QB MFL ZWV ZJ BIV JVM BIFB KQK. IVYRQZWV DYFWDVYL IFK LQRGUO YZUUVK ZHVY ZW BIV DYZXWK FWK WVHQUUVL IFKWB RZHVK FB FUU. GVYIFGL PYZZRL UQTV IZYLVL EZXUK BVUU MIVW OZX MVYV FJYFKQ BIZXDIB IFYYO BIVYV MFL F SXFHVY QW WVHQUUVL HZQEV BIFB LFQK ZWUO BZZ EUVFYUO BIFB IV MFWBVK BZ TVVG IQL JVVB ZW BIV DYZXWK. RFKFR IZZEI BIVW LIZMVK BIVR IZM BZ RZXWB BIVQY PYZZRL MQBIZXB LUQKQWD ZJJ BIV VWK FWK MFUTVK XG FWK KZMW BIV YZML EZYYVEBQWD BIVQY DYQGL.
**Exercise Set 3**

Here are the frequencies of each letter in this example:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
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<td>6</td>
<td>32</td>
<td>6</td>
<td>23</td>
<td>19</td>
<td>12</td>
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<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
<th>W</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
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</thead>
<tbody>
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<td>0</td>
<td>6</td>
<td>4</td>
<td>21</td>
<td>11</td>
<td>1</td>
<td>3</td>
<td>18</td>
<td>45</td>
<td>26</td>
<td>11</td>
<td>27</td>
<td>38</td>
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</tbody>
</table>
The ciphers that we looked at tonight clearly are not strong enough to ensure communications are secure.
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Next week we will begin our build up to another encryption scheme: RSA Encryption.