UNIVERSITY OF WATERLOO
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# Senior Math Circles

October 16, 2013

## *Problem Solving, Part 2 - Problem Set*

1. Prove that $n^2 + 3n + 5$ is never divisible by 121 for any positive integer $n$.

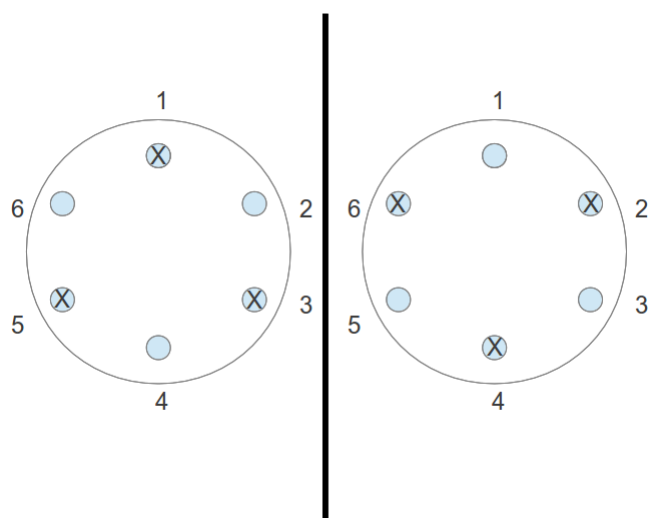
2. Prove that $n!+1$ and $(n+1)!+1$ are always relatively prime numbers, for any positive integer $n$.

3. Prove that if $ax^2 + bx + c = 0$ has real solutions for $x$ and that $a, b, c$ are not zero, then $a$, $b$ and $c$ cannot form a geometric sequence.

4. What is the largest two-digit number that becomes 75% greater when its digits are reversed?

5. Oi-Lam tosses three fair coins and removes all of the coins that come up heads. George then tosses the coins that remain, if any. Determine the probability that George tosses exactly one head.

6. Suppose there are $n$ plates equally spaced around a circular table. Ross wishes to place an identical gift on each of $k$ plates, so that no two neighbouring plates have gifts. Let $f(n, k)$ represent the number of ways in which he can place the gifts. For example $f(6, 3) = 2$, as shown below:




   (a) Determine the value of $f(7, 3)$.

   (b) Prove that $f(n, k) = f(n - 1, k) + f(n - 2, k - 1)$ for all integers $n \geq 3$ and $k \geq 2$.