Intermediate Math Circles

Rate Problems

1. Alf is in Apsley, Ontario and Barb is in Buffalo, New York, which is 360 km away from Apsley. At 9:00 a.m. Alf starts driving towards Buffalo at 80 km/h and Barb starts driving towards Apsley at 40 km/h. At what time do they pass each other, and how far away from Apsley are they at this point? (You may assume no delays in the border crossing.)

2. Wendy leaves Whitby at 5:00 p.m. and is supposed to meet her friend Wanda in Waterloo, 150 km away, at 7:00 p.m. She gets stuck in traffic for the first half hour and only manages an average speed of 30 km/h during that time. What must her average speed be for the rest of the trip in order to make it to Waterloo on time?

3. Janice’s backyard pool springs a leak and begins losing water at a rate of 2 litres per minute. After an hour Janice notices the water level is lower than usual and begins filling the pool with a hose that outputs water at a rate of 10 litres per minute. If the pool continues to leak while she is refilling it, how long will it take for the pool to be full again?

4. Emma and Julie start at the same point on a 500 m circular track and run in opposite directions at constant but different speeds. They pass each other, running in opposite directions, after 54 seconds. If Emma, who is the faster runner, completes a lap of the track in 90 seconds, how long does it take Julie to complete a lap?

5. Two candles of equal length are lit at noon. One candle takes 9 hours to completely burn while the other takes 6 hours to completely burn. At what time will the slower burning candle be exactly twice as long as the faster burning one?

6. An airplane flies at a rate of 400 km/h in still air. It can cover 900 km flying with the wind in the same time it takes to fly 700 km against the same wind. What is the speed of the wind?

7. Two people are running laps around a 400 m track. They begin at the same point and run in the same direction. The faster person runs at a pace of 1 kilometer every 4 minutes and the slower person runs at a pace of 1 kilometer every 6 minutes. How long will it take until the faster person laps the slower person?

8. James took a trip, first travelling on a train at 80 km/h and then in a car at 90 km/h. The entire trip of 265 km took 3 hours to complete. If he didn’t spend any time waiting between the train ride and the car ride, how long did he spend in each of the two vehicles?

9. Two horsemen spot each other from 400 m apart, and start riding towards each other, one at 2 \( \frac{m}{s} \) and the other at 3 \( \frac{m}{s} \). A fly starts at one horse and, flying at 8 \( \frac{m}{s} \), flies to the other horse, turns around and immediately flies back. If the fly continues flying back and forth until the horses meet, what total distance does the fly cover?

Turn the page over for the answers. Solutions will be posted online
Intermediate Math Circles

Rates I

Problem Set

Answers Only

1. Alf and Barb pass each other after 3 hours at noon, 240 km from Apsley.

2. Wendy’s average speed must be 90 km/h for the remainder of the trip to arrive in Waterloo on time.

3. Janice will refill the pool in 15 minutes (and she will still have to plug the leak).

4. Julie will run a complete lap in 135 seconds.

5. The slower burning candle will be double the height of the faster burning candle in 4.5 hours. Since they started burning at noon, this will occur at 4:30 p.m.

6. The speed of the wind is 50 km/h.

7. In 4.8 minutes or 4 minutes 48 seconds the faster runner will lap the slower runner.

8. He spent 0.5 hours in the train and 2.5 hours in the car.

9. the fly covers 640 m going back and forth from rider to rider.

Check your answers. If you have incorrect answers, try the problem again. On Wednesday night we can take a few minutes to look at problems that caused difficulty (if any).