Some problems concerning inversion in circles.

1. Consider a circle of inversion, and four straight lines: One missing the circle completely, one tangent to the circle, one cutting the cycle in two points and one going through the centre of the circle. Using compasses and straight edge, try to determine what the images of those lines are after inversion. (Use separate diagrams for each line, or your diagram will get too congested!)

2. Consider a circle of inversion and five more circles: one outside the circle of inversion, one inside the circle of inversion, one through the centre of the circle of inversion, one intersecting the circle of inversion (not orthogonally) and one orthogonal to the circle of inversion. What are their images after inversion? (Use separate diagrams for each of the 5 circles.)

3. Consider two parallel lines, with infinitely many circles between them. Circles that are each tangent to two other circles, and also to the two lines. Now invert with respect to some circle whose centre is not on either of the parallel lines. . What do you get? Do this by thinking it out. When you invert the two parallel lines what do you get?

4. Suppose I want two concentric circles so that exactly 12 circles with unit radius (i.e. radius 1) will fit in-between. What should their radii be?

   Answers: 3.864 ± 1

5. Find the radical axis for the two circles $x^2 + y^2 = 1$ and $(x - 4)^2 + y^2 = 4$. Find the two possible centres of inversion such that the circles $x^2 + y^2 = 1$ and $(x - 4)^2 + y^2 = 4$ are inverted to concentric circles. Using the centre of inversion with the smaller x-coordinate, and the circle of inversion with radius 1, find the common centre of the two circles after inversion. You might also find their radii. (The numbers get ugly, so you will need a calculator.)

   Answers:
   The radical axis is the line $x = \frac{13}{8}$.
   The two possible centres are $(\frac{13}{8} \pm \frac{\sqrt{105}}{8}, 0)$.
   The common centre of the inverted circles is (.73449,0).
   The radii of the inverted circles are .21355 and 1.13433.

6. Given the 4 circles C_1, C_2, C_3, C_4 such that the pairs C_1, C_2 and C_2, C_3 and C_3, C_4 and C_4, C_1 are externally tangent, prove that the 4 points of tangency are concyclic.