UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

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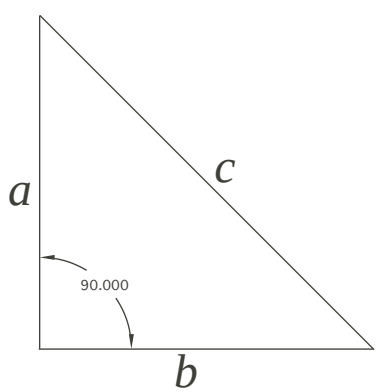
Grade 6 Math Circles
Winter 2013
Pythagorean Theorem

Pythagorean Theorem

The Pythagorean theorem is an extremely popular theorem that is used a lot in the real world to relate the side lengths of **right angled triangles**. It was uncovered by a Greek mathematician Pythagorus. This theorem states:

$$a^2 + b^2 = c^2$$

Where,



Remember that: $a^2 = a \times a \qquad b^2 = b \times b \qquad c^2 = c \times c$

And that the **longest side** on the triangle, c , is called the **hypotenuse**.

Applying Pythagorean Theorem

If $a = 3$ and $b = 4$. What is c?

We put 3 in for the value of a, and 4 in for the value of b, to get:

$3^2 + 4^2 = c^2$

$(3 \times 3) + (4 \times 4) = c^2$

$9 + 16 = c^2$

$25 = c^2$

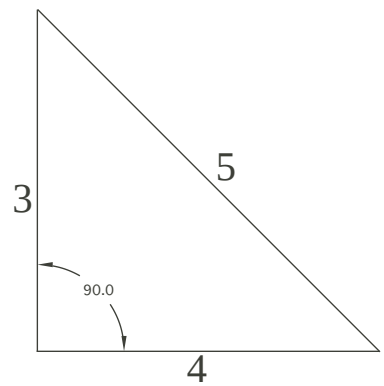
Now take the square root,

$\sqrt{25} = c$

$5 = \mathrm{c}$

<u>Note:</u> Taking the square root of a number is doing the opposite for squaring a number. If we **square** a number we are multiplying it by itself. $5^2 = 5 \times 5 = 25$. Taking the square root means finding that number we multiplied by itself to get 25, so $\sqrt{25} = 5$.

On a triangle this would mean,

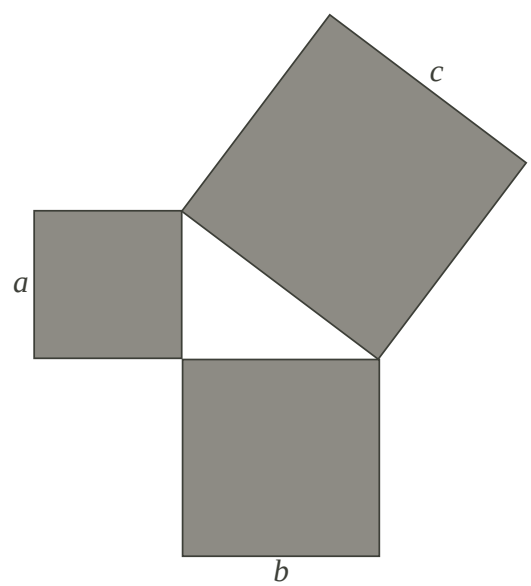


<u>Proofs</u>

There are many cool ways to prove that $a^2 + b^2 = c^2$. One way we tried physically, another we saw visually and below are two more ways.

First

Draw three squares of different sizes, and have each of them touching by one corner like so:

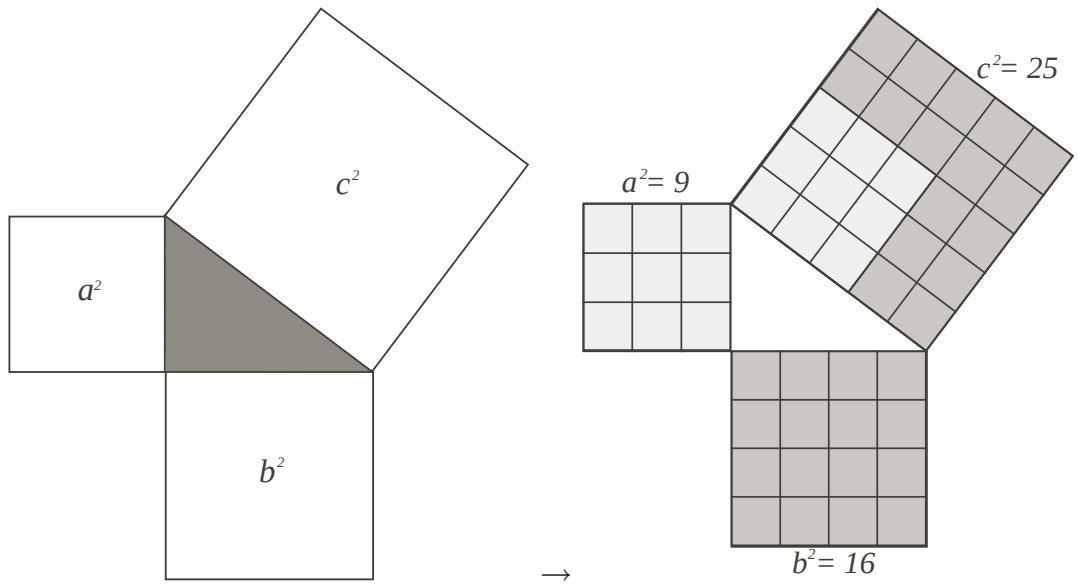


The small square has a side length of a, so it has an area of a^2.

The medium square has a side length of b, so it has an area of b^2.

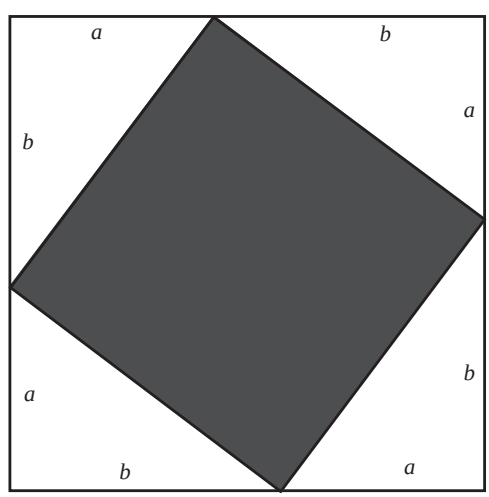
The large square has a side length of c, so it has an area of c^2.

Keep in mind, we want to show that $a^2 + b^2 = c^2$. How is this shown below?



We can see that the three squares form a right angled triangle in the middle. So we can see that the sides of a right angled triangle relate by $a^2 + b^2 = c^2$.

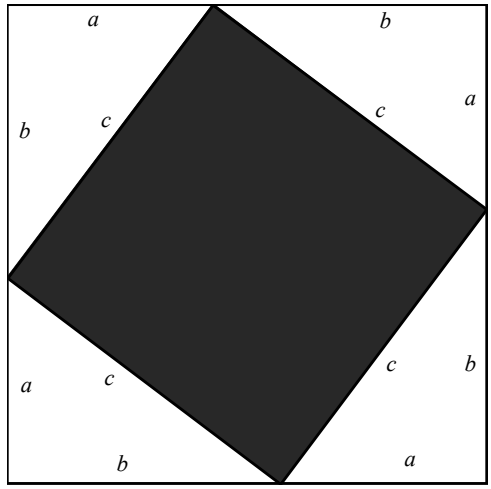
Second



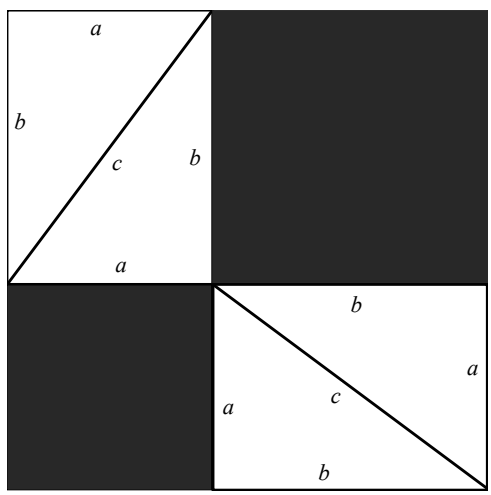
<u>Note</u>: For any two similar triangles, if two similar sides are the same length and the angle between two sides are the same in either triangle, then we can say the two triangles are the same.

Above we have four triangles positioned in a square, with a dark region in the middle.

We can see that all four triangles have equal sides a and b, and so the angles between a and b will be equal in all four triangles. This means we have four equal triangles, so the hypotenuse of these triangles will all be equal. Let's say the hypotenuse has a length of c.



If we take the area of the dark region $= c \times c = c^2$



If we move over two triangle to make 2 rectangles out of the triangles, we end up with another two more dark regions:

1. one with area $a \times a = a^2$
2. another with $b \times b = b^2$

What do we see? $a^2 + b^2 = c^2$!

Pythagorean Triples

A Pythagorean triple is a set of 3 numbers that have perfect solutions to the Pythagorean Theorem. The easiest triple is:

3 4 5

We can take **multiples** of this to make more, like:

6 8 10
9 12 15
12 16 20

But there are also other Pythagorean triples that **aren't** multiples of the 3 4 5:

5 12 13
7 24 25
9 40 41
11 60 61

What can we notice?

1. the 2nd and 3rd number differ by 1
2. the 2nd + 3rd number = the 1st number square

a	b	c	a^2	$\frac{a^2}{2}$
3	4	5	9	4.5
5	12	13	25	12.5
7	24	25	49	24.5
9	40	41	81	40.5
11	60	61	144	60.5

Knowing this, how do we generate a Pythagorean triple whose first number is 13?

Solution:

$13^2 = 169$

$\frac{169}{2} = 84.5$

$84 + 85 = 169$

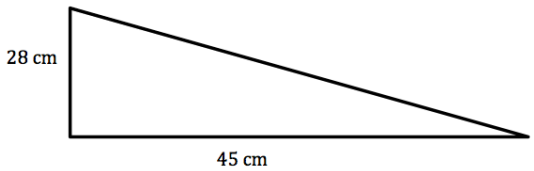
So the Pythagorean triple for a 1st number of 13 is : 13 84 85

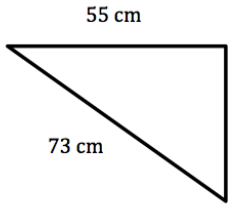
Problem Set

1. What is the value of c if:

   a) if $a = 17$ and $b = 144$
   b) if $a = 35$ and $b = 612$
   c) if $a = 8$ and $b = 15$
   d) if $a = 20$ and $b = 21$

2. Calculate the missing side length.

   a) 

   b) 

3. Find the triple that contains a first number of 23.

4. Which of the following is NOT a pythagorean triple?

   a) 19 180 181
   b) 35 612 613
   c) 40 76 86
   d) 12 35 37

5. Which of the following is a pythagorean triple?

   a) 6 13 14
   b) 20 21 29
   c) 7 13 15
   d) 10 16 19

6. If Brampton is 21 km north of Mississauga, and St Thomas is 220 km west of Mississauga, what is the distance between Brampton and St Thomas?

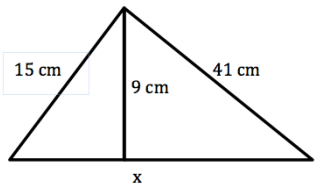
7. If the length of a rectangle is 16 cm long, and diagonal is 34, what is the width?

8. A man props a ladder of 65 m against a building so he can get to the roof. The distance from the foot of the ladder to the building is 33 m. How tall is the building?

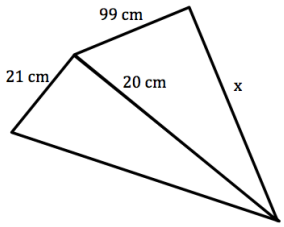
9. A baseball diamond is a square with sides of 72 m. What is the distance between first base and third base?

10. Find the value of x.

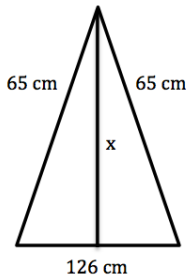
a)



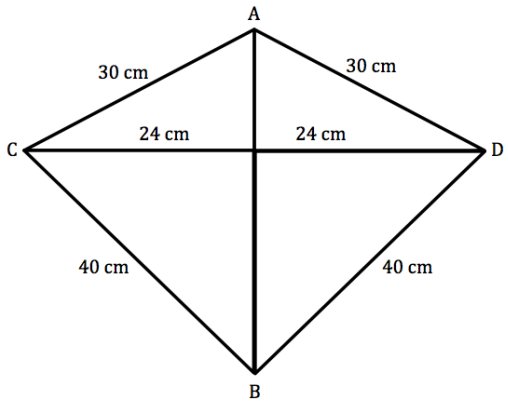
b)



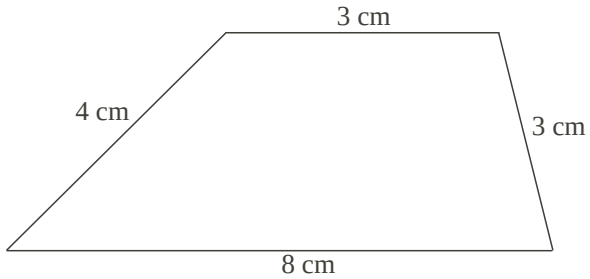
c)



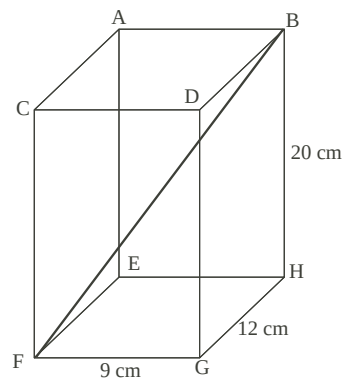
11. Find the length of AB.



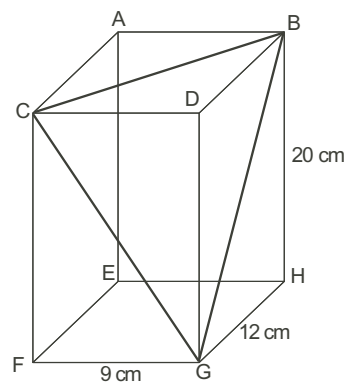
12. Find the area of the trapezoid below using all the information give.



13. Given the rectangular prism below, find the length of the line BF.



14. Given the same rectangular prism below, find the area of the triangle within the prism, as formed by the lines CB, CG and GB.



15. A square-based pyramid has a height of 8 m and base area of 144 m^2. What is the length of the slant of the pyramid?

16. A semi-circle is half of a circle. Using three semi circles, to prove in another way that $a^2 + b^2 = c^2$. (Hint: the area of a semi-circle is $\frac{\pi r^2}{2}$.)

Solutions

1. a) 264 265 b) 613 c) 17 d) 29

2. a) $\sqrt{2809} = 53$; b) $\sqrt{2304} = 48$

3. 144 145

4. c)

5. b)

6. 221 km $= \sqrt{48841}$

7. $30 = \sqrt{900}$

8. $\sqrt{3136} = 56$

9. $\sqrt{10368}$

10. a) 52; b) 101; c) $\sqrt{256} = 16$

11. 50 cm

12. 13.2

13. 25 cm $= \sqrt{625}$

14. CB = 15 cm; CG $= \sqrt{481}$;BG $= \sqrt{544}$. Area of triangle $= \dfrac{b \times h}{2} = \dfrac{15}{2}$

15. $\sqrt{136}$

16. From the diagram below, you can caluculate and see that the area of the small semi-circle, and the area of the medium semi-circle equals the area of the large semi-circle.