Intermediate Math Circles  
Analytic Geometry II  
Problems February 28, 2013

1. Prove, using analytic methods, that the diagonals of a parallelogram bisect each other.

2. Prove, using analytic methods, that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and one-half the length of the third side.

3. Determine the distance from the point $Q(-3,5)$ to the line $2x - 7y + 1 = 0$.

4. Calculate the coordinates of the foot of the perpendicular from the point $(2,-6)$ to the line $x - 3y - 2 = 0$.

5. In $\triangle ABC$, with vertices $A(2,1)$, $B(12,6)$ and $C(0,10)$, an altitude is drawn from $C$ touching $AB$ at $D$. Determine the length of the altitude $CD$. Develop two different solutions.

6. A point $P$ is chosen on the line $y = 2x + 3$ and a point $Q$ is chosen on $y = -x + 2$. If the midpoint $M$ of the line segment $PQ$ is $(2,5)$, determine the coordinates of $P$ and $Q$.

Answers

3. $\frac{40}{\sqrt{53}} = \frac{40\sqrt{53}}{53} \approx 5.5$

4. $\left(\frac{1}{5}, \frac{-3}{5}\right)$

5. $4\sqrt{5}$

6. $P(3,9)$ and $Q(1,1)$