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Centre for Education in
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Grade 11/12 Math Circles
Symbolic Logic I Exercises
October 31, 2012

1. As discussed, the word "or", as interpreted logically, is the *inclusive* "or", meaning that $(P \vee Q)$ will be true in any of the cases when P is true or Q is true, and even when P and Q are both true. Write out the *exclusive* "or" in terms of the logical connectives (that is, make sure that $P \vee Q$ will be false if both P and Q are true).

2. Write the truth tables for each expression.

   (a) $\sim (\sim P)$

   (b) $P \Rightarrow (Q \vee R)$

   (c) $(P \wedge Q) \Rightarrow R$

   (d) $(P \vee \sim Q) \Rightarrow R$

   (e) $\sim P \Rightarrow (Q \Leftrightarrow R)$

3. Using truth tables, prove that the statements $\sim (P \vee Q)$ and $\sim P \ \wedge \sim Q$ are equivalent. Give an example of the equivalence of these statements in everyday language.

4. Denote the "exclusive or" described in Problem 1 by "XOR". Using truth tables, prove that
$$(P \text{ XOR } Q) \Leftrightarrow \sim (P \Leftrightarrow Q).$$

5. Let P be the statement "It is snowing" and let Q be the statement "It is freezing". Write each statement using P, Q and connectives.

   (a) If it is snowing, then it is freezing.

   (b) It is freezing but not snowing.

   (c) When it is not freezing, it is not snowing.

6. Let P be the statement "It is raining", let Q be the statement "I ride my bike to school", let R be the statement "I have an umbrella", and let S be the statement "I can walk to school". Express each statement as an English sentence.

   (a) $\sim P \Rightarrow Q$

   (b) $P \Rightarrow \sim Q$

   (c) $(P \wedge R) \Rightarrow S$

   (d) $(P \wedge \sim R) \Rightarrow \sim (Q \vee S)$

7. Imagine that a psychologist lays four cards on a table in front of you. He tells you that each card has a number on one side and a letter on the other. On the uppermost faces of the cards you see the four symbols

   $$E \qquad K \qquad 4 \qquad 7$$

   The psychologist then tells you that the cards are printed according to the rule: *If a card has a vowel on one side, it has an even number on the other side.* He then asks you which cards do you *have* to turn over to be sure that all four cards satisfy the rule. How do you answer?