



Grade 7/8 Math Circles
April 4, 2012
Math Olympics

Speedy Arithmetic:

100 points: $1469273 \times 11 = ?$

200 points: How many values are there between 928 and 1024 that are divisible by 9?

300 points: $\frac{1}{2} \times \frac{3}{2} \times \frac{12}{6} \times \frac{2}{4} \times \frac{80}{100} \times \frac{10}{4} \times \frac{1}{3} \times = ?$

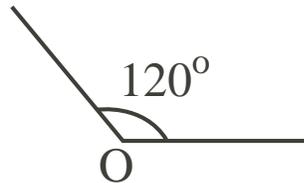
400 points: For what value(s) of p and q is $2p346q$ divisible by 6?

500 points: Prove the divisibility rule for 3 for a 4-digit number. (i.e. A (4-digit) number is divisible by 3 if and only if the sum of its digits is divisible by 3).

Ancient Geometric Constructions:

100 points: Who is the ancient Greek mathematician is widely known as the father of geometry?

200 points: Make a 60° angle using the diagram below:

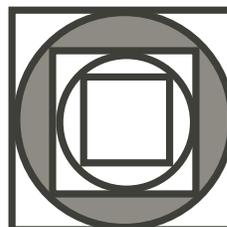


300 points: Copy this line: (the resulting copy's orientation doesn't matter)



400 points: With its sides being whole numbers, what is the smallest possible perimeter of a non-equilateral triangle?

500 points: The smallest square has area 32. What is the area of the shaded region in the diagram below?



Algebraic Approaches:

100 points: The average of 4 numbers is 20. One of the numbers is 18. What is the sum of the other 3 numbers?

200 points: In a science experiment that involves looking at the growth of plants, we are told that plants require liquid, soil and light to grow. If you are given 5 kinds of liquid, 3 kinds of soil and 2 kinds of light, How many plants do you need to plant at minimum to guarantee that at least 2 out of all the plants grow under exactly the same conditions?

300 points: Find the value of x .



400 points: It's spring time, and that means... STRAWBERRY PICKING! Laura can fill a basket with strawberries in 3 hours; Grace on the other hand goofos around when she picks strawberries and can fill the same kind of basket in 5 hours. How long would it take them together to fill the basket?

500 points: How many positive integers n , with $n \leq 100$, are there such that $n^3 + 7n^2$ is a *perfect square*? (A *perfect square* is a number whose square root is an integer, for example, 1, 4, 9, etc...)

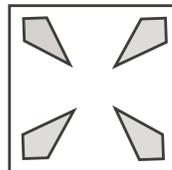
Spatial Visualization and Origami:

100 points: A fold that could be pressed flat on a surface (i.e. 2D in a strict sense) is called _____. It is ____-colourable.

200 points: The picture below shows Sally with a very festive hair-do and make up as seen in the mirror. When Sally is facing you, what does she look like?

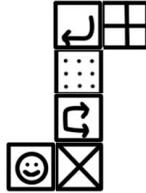


300 points: How many times does the paper need to be folded to get the pattern in the diagram below down to its *smallest component*? What does its smallest component look like? (recall that a *smallest component* is the portion of a whole pattern such that it is no longer symmetrical in any way).



400 points: A square sheet of breakfast crepes is folded 3 times, creating an isosceles triangle each time. If a bite is taken out of the 90° angle such that the bite mark parallels the hypotenuse. What does my unfolded crepe look like?

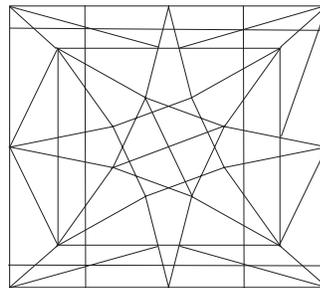
500 points: The diagram below is a construction diagram of a cube. After constructing, is this configuration possible?



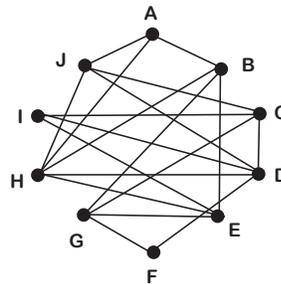
Introduction to Graph Theory:

100 points: Define a path and a cycle.

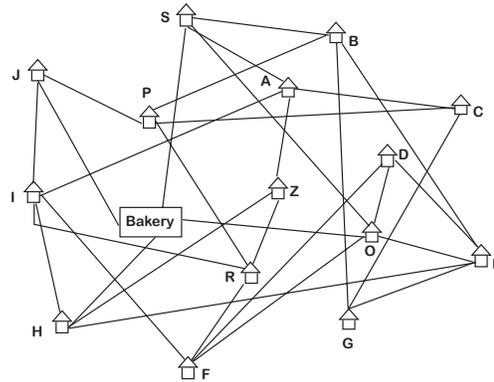
200 points: Properly colour the graph below using at most 4 colours.



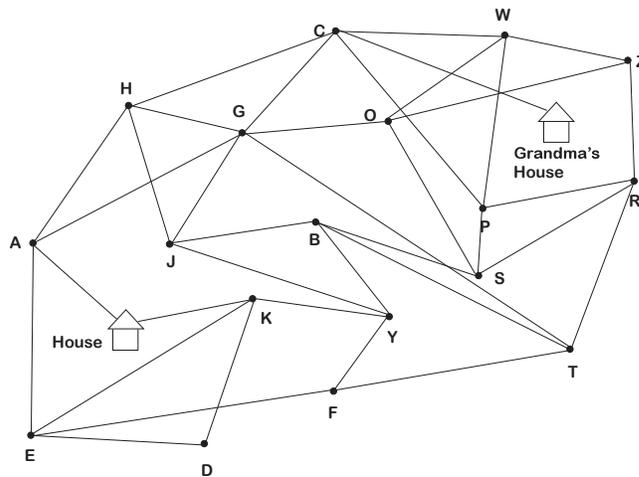
300 points: Determine a path from vertex A to F using the graph below that includes every vertex.



400 points: Bessy the baker needs to deliver 15 cakes to 15 different houses. Using the graph below, determine a route that Bessy should take from her bakery, to each house, and return to the bakery without taking the same road or visiting the same house twice.



500 points: Little Red Riding Hood needs to walk through the forest to deliver a basket full of food to her Grandma's house. She has discovered that if she passes through every checkpoint (vertex) on the graph below, she is able to avoid the Big Bad Wolf. Determine a path that Red Riding Hood should take to Grandma's House without visiting the same checkpoint or walking along the same path twice.

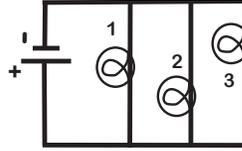


Circuits:

100 points: State Ohm's law and the properties for current, voltage and resistance for series and parallel circuits.

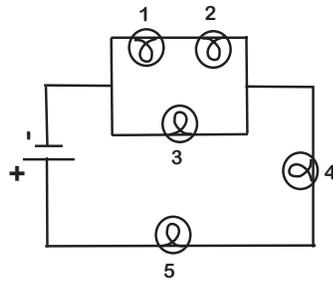
200 points: A 6V battery is connected to a single light bulb in a circuit. If the resistance at the light bulb is measured to be $2\ \Omega$, what is the current passing through the light bulb?

300 points: For the diagram and corresponding table below, fill in the missing values in the table using Ohm's Law and circuit properties.



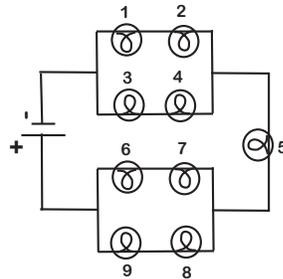
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$
$I_{total} =$	$I_1 =$	$I_2 = 2\text{A}$	$I_3 =$
$R_{total} = 2\Omega$	$R_1 = 4\Omega$	$R_2 = 8\Omega$	$R_3 =$

400 points: For the diagram and corresponding table below, fill in the missing values in the table using Ohm's Law and circuit properties.



$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 =$
$I_{total} = 1\text{A}$	$I_1 =$	$I_2 = \frac{2}{3}\text{A}$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 = 1\Omega$	$R_2 = 2\Omega$	$R_3 = 6\Omega$	$R_4 = 3\Omega$	$R_5 = 5\Omega$

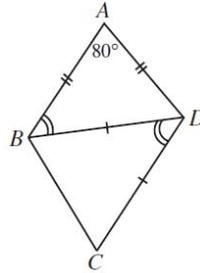
500 points: For the diagram and corresponding table below, fill in the missing values in the table using Ohm's Law and circuit properties.



$V_{total} = 36\text{ V}$	$V_1 =$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 = 6\text{ V}$
$I_{total} =$	$I_1 = 2\text{ A}$	$I_2 =$	$I_3 =$	$I_4 = 1\text{ A}$	$I_5 = 3\text{ A}$
$R_{total} = 12\Omega$	$R_1 = 4\Omega$	$R_2 = 2\Omega$	$R_3 = 8\Omega$	$R_4 = 4\Omega$	$R_5 =$
	$V_6 =$	$V_7 =$	$V_8 =$	$V_9 =$	
	$I_6 =$	$I_7 = \frac{3}{2}\text{ A}$	$I_8 = \frac{3}{2}\text{ A}$	$I_9 =$	
	$R_6 = 8\Omega$	$R_7 = 4\Omega$	$R_8 = 9\Omega$	$R_9 = 3\Omega$	

Gauss Contest:

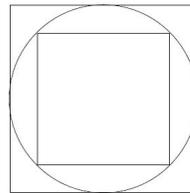
100 points: In the diagram below, $\angle ABD = \angle BDC$ and $\angle DAB = 80$. Also, $AB = AD$ and $DB = DC$. What is the measure of $\angle BCD$?



200 points: Chloe has made a code out of the alphabet by assigning a numerical value to each letter. She then assigns a numerical value to a word by adding up the numerical values of the letters in the word. Using her code, the numerical value of BAT is 6. Also, her code gives numerical values of 8 to CAT and 12 to CAR. Using her code, what is the numerical value of BAR?

300 points: In Math Idol, there was a total of 5 219 000 votes cast for four potential Idols. The winner received 22 000 more votes than the 2nd place contestant, 30 000 more than the 3rd place contestant, and 73 000 more than the 4th place contestant. How many votes did the winner receive?

400 points: In the diagram below, a circle is inscribed in a large square and a smaller square is inscribed in the circle. If the area of the large square is 36, what is the area of the smaller square?



500 points: A large block, which has dimensions n by 11 by 10, is made up of a number of unit cubes and one 2 by 1 by 1 block. There are exactly 2362 positions in which the 2 by 1 by 1 block can be placed. What is the value of n ?

Mystery:

100 points: For their project, a group of students needed 30 poles, each 6 units in length. They also needed 20 poles, 8 units in length, and ten poles, 10 units in length. The poles were only available in 16 unit lengths. Assuming no gluing and no loss in cutting, what is the least number of 16 unit length poles needed?

200 points: Harry, Ron, Hermione, and Ginny are trying to fly from Hogwarts castle to the Weasley's house but they only have one broom. The broom can only hold one of Harry and Ron or both (or one) of Hermione and Ginny. What is the minimum number of trips between Hogwarts and the Weasley's house needed to move all four people from Hogwarts to the Weasley's house?

300 points: Tony and Maria are training for a race by running all the way up then back down a 700m long ski slope. They each run at different but constant speeds. When they run back down the slope, each person runs at double his or her uphill speed. Maria reaches the top first, and immediately starts running back down the hill, meeting Tom 70m from the top. When Maria reaches the bottom of the hill, how far behind is Tom?

400 points: Joshua, Cassy, and Emily divide a sum of money that they won from the lottery. The sum of money was divided as follows:

- Joshua receives \$775 plus $\frac{1}{5}$ of what remains;
- Cassy receives \$900 plus $\frac{1}{6}$ of what remains; and
- Emily receives the rest, which is \$1000.

How much was the original sum of money?

500 points: AB is the diameter of the semicircle in the diagram below. If $AC = 8$, $CB = 6$, and $\angle ACB = 90^\circ$, find the area of the shaded region.

