UNIVERSITY OF WATERLOO
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Faculty of Mathematics

Centre for Education in
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Grade 6 Math Circles
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Measurement and Geometry
Solutions to the Exercises

1. (a) By the Pythagorean Theorem, the length of the diagonal of the farmland is $250m$. Therefore, he walks $100m$ more in the summer than in the winter in a single trip from the barn to the house.

   (b) The farmer walks $100 \times 2 \times 70 = 14000m = 14km$ more in the summer days than winter days.

2. (a) The area of the big circle has radius $r = 5$, and thus has area 25π. The shaded region has 20% of the big circle's area, and thus has area $25\pi \div 5 = 5\pi$

   (b) The measure of $\angle AOB$ is $\frac{1}{5}$ of $360°$. Thus $\angle AOB = 360 \div 5 = 72°$

3. (a) A naive way to do this is to find the length of all the sides and add them up. We have our total $perimeter = 6 + (14 - 4)4 + 4 + (6 + 4) + 14 = 48$.

   (b) In this question, there are two sides whose specific length we cannot determine, namely the bottom two sides. But they add up together to 14, so we've $perimeter = 15 + 8 + 15 + 5 + (8 + 5) = 56$.

4. (a) Remember that the circumference and area of a semicircle is half of that of a full circle with the same radius. The diameter of the semicircle is $4 \div 2 = 2$We have $perimeter = 4 + 7 \times 2 + 4\pi = 18 + 4\pi$.

   (b) The radius of the semicircle is 2. We've $area = 4 \times 7 + 2^2\pi = 28 + 4\pi$.

5. $Area = \frac{(12+6)\times 3}{2} = 27$

6. $Area = 18 \times 8 = 144$

7. This is a square with side length 12, since $12 \times 12 = 144$.

8. The big square has side length 8, and thus has area $8 \times 8 = 64$. So the 3 white triangles have area $\frac{3\times 2}{2} = 3$, $\frac{5\times 8}{2} = 20$, and $\frac{6\times 8}{2} = 24$. Thus the area of the shaded regions is $64-3-20-24 = 17$.

9. The larger square has side length$48 \div 4 = 12$, the smaller square has side length $32 \div 4 = 8$. Therefore, the area of the larger square is $12 \times 12 = 144$, the area of the smaller square is $8 \times 8 = 64$. The area of the shaded region is $144 - 64 = 80$.

10. (a) Large square $ABCD$ has side length $72 \div 4 = 18$. So the base of the trapezoid T is 18. Its top is the same as the side length of one of the squares $S1$ or $S2$, which is 8. Its height is $18 - 8 = 10$. Hence the trapezoid has area $\frac{(18+8)\times 10}{2} = 130$.

    (b) We use Pythagorean Theorem to determine the length of the two sides of the trapezoid. The left side has length $\sqrt{2^2 + 10^2} \approx 10.2$, the right side has length $\sqrt{8^2 + 10^2} \approx 12.8$. Hence the perimeter of the trapezoid is approximately $8 + 18 + 10.2 + 12.8 = 49$.

11. This can be determined by using the Pythagorean Theorem twice. Draw a line down the top left corner to the bottom right corner, you fill find that you get 2 triangles. Checking using the Pythagorean Theorem verifies that both of these triangles are right angled triangles. The area of this quadrilateral is $\frac{12\times 9}{2} + \frac{12\times 20}{2} = 174$.

12. $9 = \frac{6\times x}{2}$, $x = 18 \div 6 = 3$.

13. This problem is meant to challenge those who desire a challenge :) Hint: What can you say about the height of the triangle HOP, if OP is the base? The final answer is $72 - 9 = 63$.