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Grade 6 Math Circles
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Math Magic

Solutions

Example Set 1:

1. (a) Let c be the value of the third die. If we decided to roll three die, our algebra would look like this:

$\underline{x}$
$\underline{a+b+c}$
$\underline{(7-a)+(7-b)+(7-c)}$

And the sum of these three lines would be:
$x+a+b+c+7-a+7-b+7-c$
$= x+21$

This means we would have to subtract 21 at the end to find the beginning number instead of 14.

Similarily if we had four dice we would have to subtract 28 at the end and if we had 5 dice we would have to subtract 35 at the end.

(b) Each die we add, we have to subtract 7 more at the end. Another way of looking at it is we subtract $(7 \times y)$ at the end where y is the number of dice that is used for the trick. Once we notice this, to find out how the trick would change for fifteen dice we just have to calculate $(7 \times 15) = 105$.

Example Set 2:

1. We will look at the algebra that is done for each step:

   (a) Give each digit a variable name from left to right a, b, c, and d (i.e. so the number in the ones position is d)

   (b) This digit is a

   (c) These digits are ab

   (d) These digits are abc

   (e) For this step, we have to remember that for a number to be in the tens column (say a), it is the same as writing $10 \times a = 10a$. Looking at Step (c) for example, this number is really just $10a + b$, and the number in Step (d) is just $100a + 10b + c$. We now add up the numbers in Steps (b)-(d) to get the following equation:

   $$(a) + (10a + b) + (100a + 10b + c) = 111a + 11b + c$$

   (f) That is:

   $$9(111a + 11b + c)$$

   This is where it may get tricky. Remember when you multiply 9 into a bracket, everything in the bracket must be multiplied by 9. That is:

   $$(9 \times 111a) + (9 \times 11b) + (9 \times c) = 999a + 99b + 9c$$

   (g) The sum of all the digits is just:

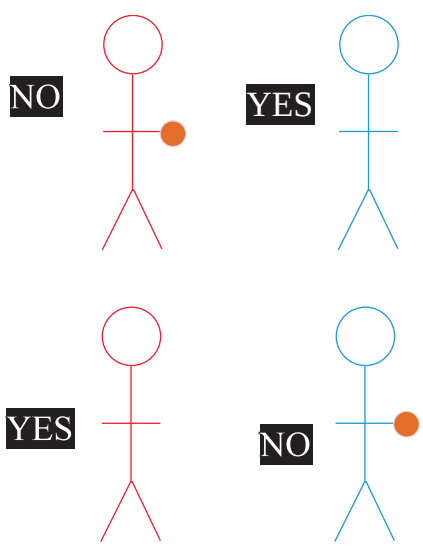
   $$a + b + c + d$$

   (h) $(999a + 99b + 9c) + (a + b + c + d) = 1000a + 100b + 10c + d$

   We now notice that this is just our magic number written in the form we mentioned in Step e) of the solutions!

2. We will look at the algebra that is done for each step:
   (a) Let's call the month m and the day d
   (b) $5m$
   (c) $5m + 7$
   (d) $4(5m + 7) = 20m + 28$
   (e) $20m + 28 + 13 = 20m + 41$
   (f) $5(20m + 41) = 100m + 205$
   (g) $100m + 205 + d$
   (h) $100m + 205 + d - 205 = 100m + d$
   (i) We now notice that this, similar to the reasoning in the previous example places the month in the hundreds column and the day is put in the ones column (it also is in the tens column since most birthdays are not in the first nine days of the month)

3. For this question, we will draw a diagram with the possible outcomes and deduce our answer using logic. Let the person who is telling a lie be RED and the person who is telling the truth be BLUE. There is two possibilities that we must look at, when the person telling the lie is holding the penny and when the person telling the truth is holding the penny. To show this, a penny is drawn in the hand of the person who is holding it for each possibility.



Notice that no matter who is holding the penny, if they say that they are not holding the penny, then they are and if they say that they are holding the penny then they are not. This is why asking the question "Is the liar holding the penny?" works.