University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
March 9, 2011
Combinations

Review

1. Evaluate 6!
2. Evaluate $\frac{7!}{5!}$
3. Assuming a "word" is just an arrangement of letters. How many 3-letter "words" are there?
4. How many 3-letter "words" are there with no repeating letters?
5. How many 3-letter "words" are there with at least one repeating letter?
6. 10 swimmers compete in a race. How many arrangements of first, second and third place are there assuming there is no tie?
7. You are judging a talent show with 9 contestants. For the first half of the show, you need to pick the order of the first four acts. How many possible ways are there?

General Rule: The number of permutations of n taken k items at a time is

$$\frac{n!}{(n-k)!}$$

The short form of this notation is ${}_n\mathrm{P}_k$

Permutations with Identical Elements

So far, we have seen permutations of items that are distinct. What happens when there are identical elements in our set?

Problem: You are making a bracelet and need to put 4 beads in a line. You have 2 red beads, 1 green beads, and 1 blue bead. How many different ways can you make the line?

Problem: How many ways can you arrange three A's and two B's?

Exercises:

1. Your weekly chores include washing the dishes 3 times, taking out the garbage once and cleaning your room twice. How many ways can you order these tasks?

2. Instead of having 4 beads, you now have 10 beads: 3 yellow, 3 green, 1 blue, 1 orange, 1 red and 1 pink. How many ways can you line all the beads up for the bracelet?

General Rule: The number of permutations of n items with a identical items of one kind, b identical items of another kind, etc. is

<u>Combinations: Order Doesn't Matter!</u>

Problem: In a singing competition, there are 5 finalists (A, B, C, D and E). As a judge, you only pick the *top three* and do not specify their placing. How many possible outcomes could there be?

In the above problem we had to choose 3 people out of a group of 5 and did not care about the order. If order mattered, then we know the answer is just $\frac{5!}{2!}$. But when order matters, we are double counting occurrences (we counted ABC as well as ACB, BAC, BCA, CAB and CBA). For any group of 3, there are $3! = 6$ groups that contain the same elements, but just in a different order. In a the case where order does not matter, we have to divide the permutation by this so that we don't double count.

Example: You just bought 7 paintings and want to put 3 of them on a wall. How many ways can you do this?

Notation: In general, if we want to choose k elements from a group of n, we know that there are $\frac{n!}{k!(n-k)!}$ ways of doing so. We can make up a short hand to represent the number of ways to choose k elements from a group of n.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

We say $\binom{n}{k}$ as "n choose k".

Problem Set

1. On an announcement board, you take out the letters to the word "CAUTION". How many ways can you arrange the letters and put them back on the board?

2. On an announcement board, you take out the letters to the word "WARNING". How many ways can you arrange the letters and put them back on the board?

3. How many ways can you order the letters of "MISSISSIPPI"?

4. From the top 40's chart, you want to pick 3 songs to move to your playlist. How many different combinations are there?

5. You have a deck of cards. How many possible five card hands can you draw?

6. With a deck of cards, a full house is a 5 card combination that has 3 of a kind and a a pair. How many possible full houses are there?