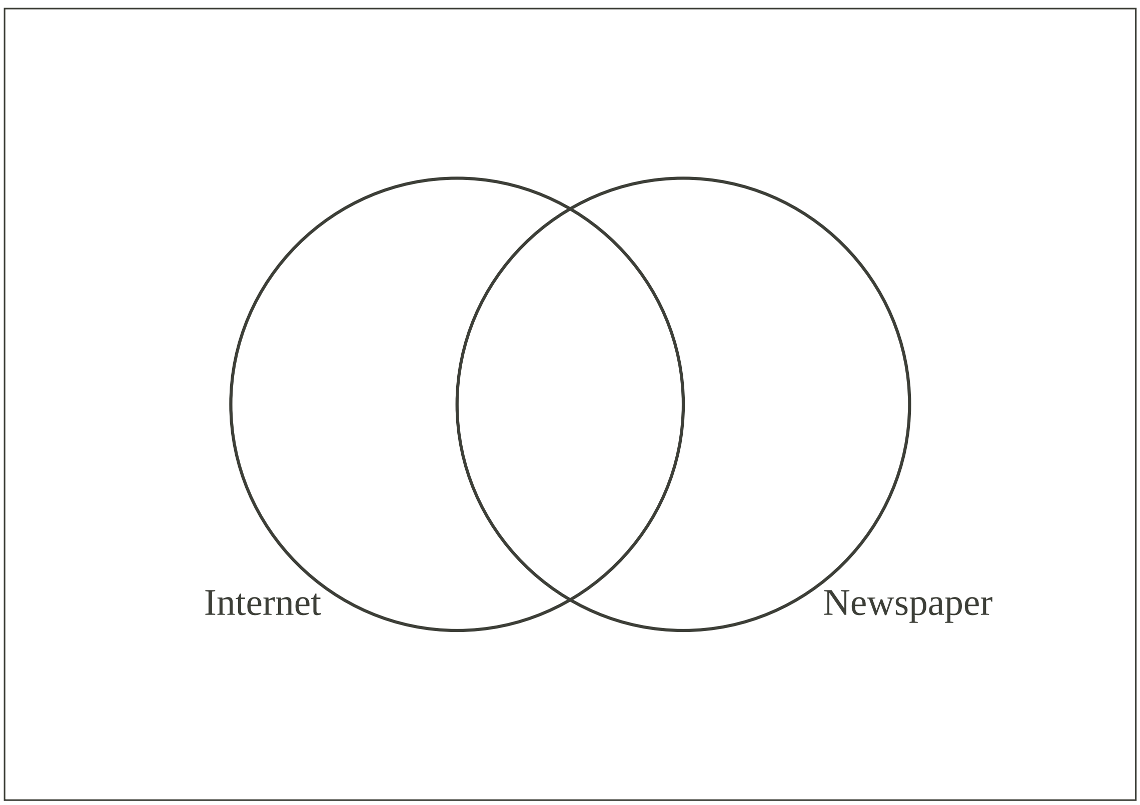
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

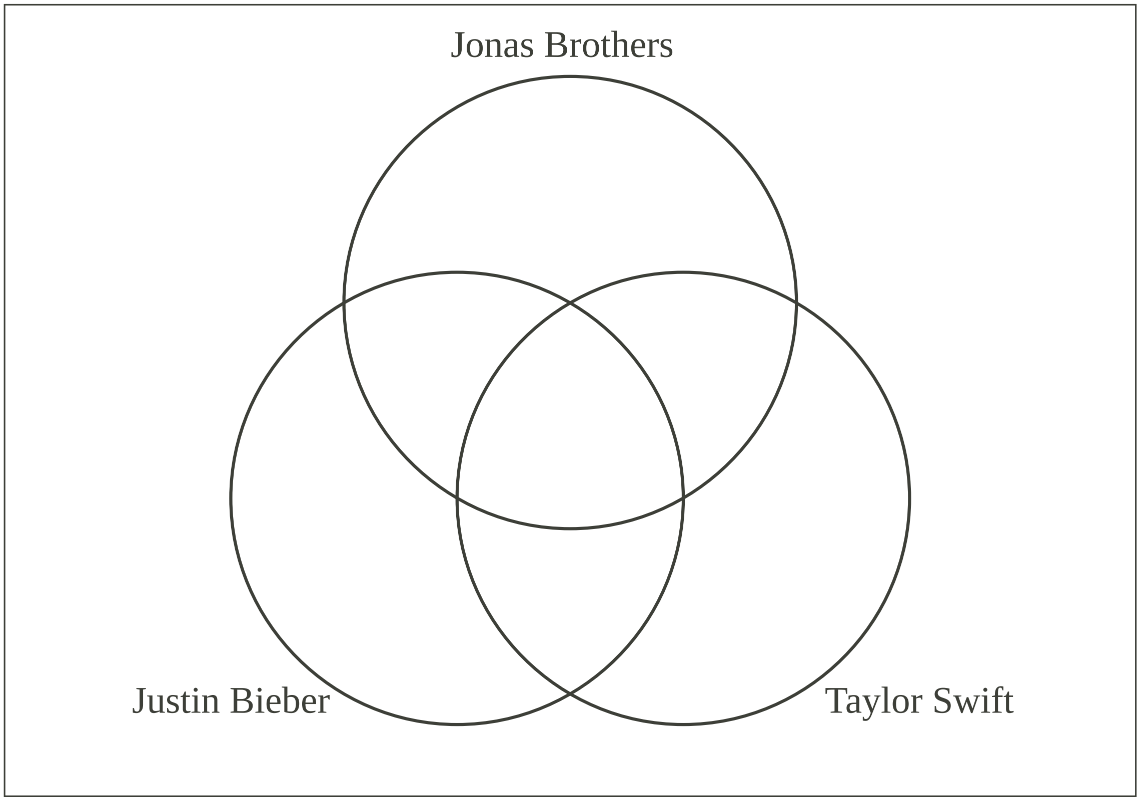
Grade 6 Math Circles
March 2, 2011
Counting

Venn Diagrams

Example 1: Ms. Daly surveyed her class of 26 students to see how they read about the news. 6 of them read the newspaper, and 15 of them read news from the internet. If 4 of the students read both the newspaper and from the internet, how many students in Ms. Daly's class read news from the internet **or** newspaper? How many students don't read from either of these two sources?



Example 2: Ms. Iockis surveyed her class of 24 students about the music they like. Fill in the Venn diagram and answer the questions.



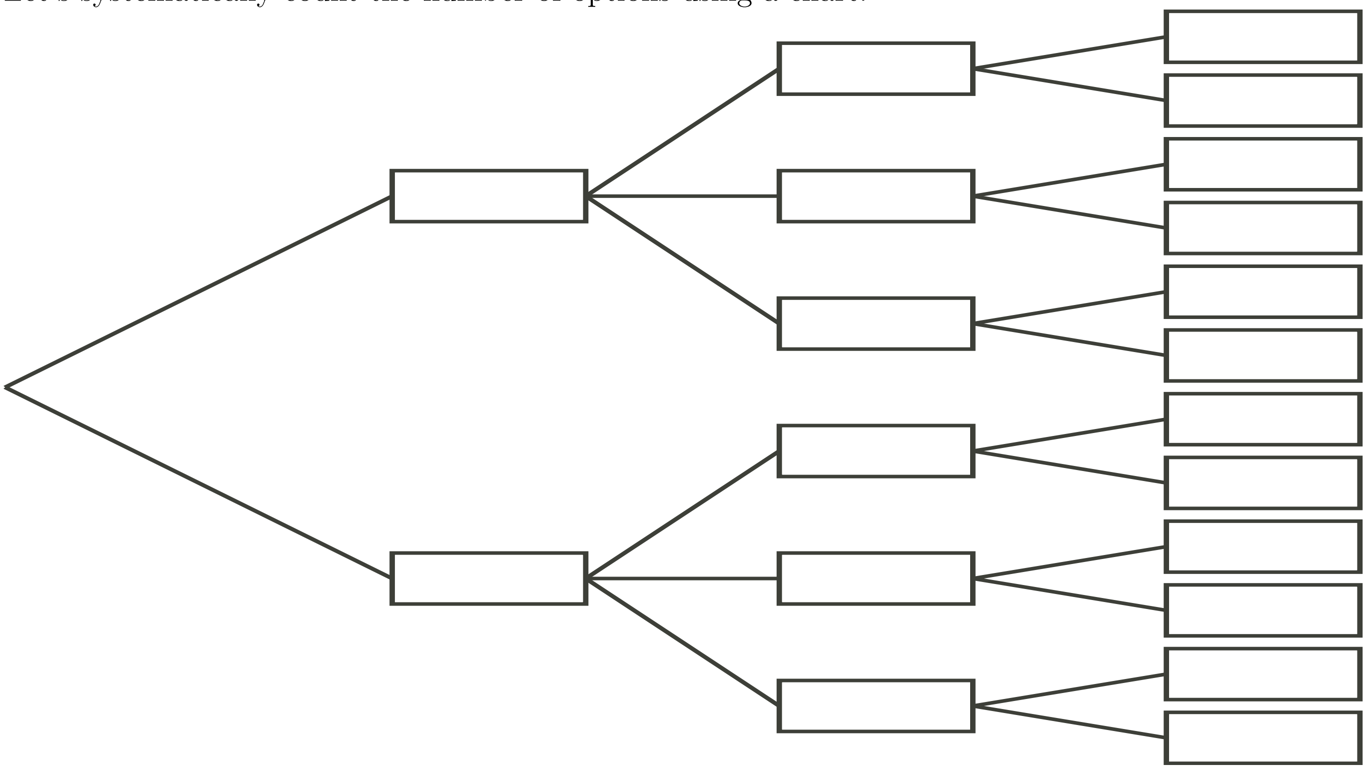
- 15 students like Taylor Swift
- 10 students like Justin Bieber
- 12 students like Jonas Brothers
- 3 students like Justin Bieber and Taylor Swift, but not Jonas Brothers
- Half of the students who like Jonas Brothers also like Justin Bieber
- 20 students like Jonas Brothers or Taylor Swift (or both)
- 8 students like Taylor Swift but not Justin Bieber

How many students don't like any of the three artists?
How many students like Jonas Brothers or Justin Bieber?
How many students like Jonas Brothers but not Taylor Swift?
How many students like Taylor swift but not Jonas Brothers?

Fundamental Counting Principle

Problem: You and your family are ordering the Winter special at a restaurant. The menu let's you choose one of two mouth-watering appetizers (soup of the day or salad), one of three hot and juicy entrees (beef steak, salmon or chicken) and one of two scrumptious desserts (cake or sundae). How many possible ways can you have your meal?

Let's systematically count the number of options using a chart.



What if there are 10 different appetizers, 15 different entress and 4 different desserts?

This leads us to the **Fundamental Counting Principle** (or the "multiplication rule"), which states: If we have m ways of doing something and n ways of doing some other thing, then there are mn ways of doing both actions.

Exercises:

1. Danny has 3 pairs of glasses, 4 hats and 2 nose rings. He can only wear one of each kind of accessory at a time. How many different "looks" can he have before he has to repeat a "look"?

2. Jillian has a purple six-sided die (each face numbered 1 through 6) and a yellow coin (heads on one side and tails on the other). She flips the coin and rolls the die. How many different possibilities should she expect?

3. A robber discovers a safe with a number pad (0 to 9). The safe opens when the correct 4-digit password is entered. How many possibilities are there?

4. You take a coin, flip it and record which side faces up. You continue doing so, recording which side faces up as well as the trial number (e.g. trial 1, trial 2, etc...). How many possibilities in this experiment are there if there are 3 trials? 10 trials? n trials?

5. You number five balls 1 to 5. and put them in a box. You randomly pick out a ball, record the number then put it back in and repeat the process three times. How many possible sequences do you have?

Permutations: Order Matters!

A **permutation** is an arrangement of a certain number of things. For example, if you want to arrange three letters $A, B,$ and C, a CAB is one permutation and ABC, is another permutation. A common question to ask is "given a certain set of things, how many permutations are there?"

Problem: Allie, Bo, Cindy and David are lining up to buy tickets for a basketball game. How many different ways can they line up?

Q: Is there some systematic way to explain this?
A: Yes there is!!

Factorial Notation: Consider the line up example, but instead of having four people, we have 100 people!! Calculating that even with our method is going to be difficult. Thankfully, we have a notation for this kind of multiplication called the **factorial**.

$$n! = n \times (n-1) \times (n-2) \times ... \times 3 \times 2 \times 1$$

The "!" is the factorial notation, and we say "n!" as "n factorial".

Example: Evaluate 5!

Exercises:

6. You are stacking 6 distinct coins on top of each other. How many different arrangements can you have?

7. You need to do laundry, cook, do homework and hang out with friends. Assuming you cannot multitask, how many ways can you order your tasks?

Problem Set

1. (a) How many permutations are there of the letters of the word *guitar*?

   (b) How many of those permutations start with g?

   (c) How many of the permutations in part (a) have t and r together?

2. How many 4-digit numbers are there with no repeating digits? *(Remember: numbers starting with 0 don't count!)*

3. How many 4-digit numbers are there with at least a digit repeated? *(Hint: how many 4 digit numbers are there in total?)*

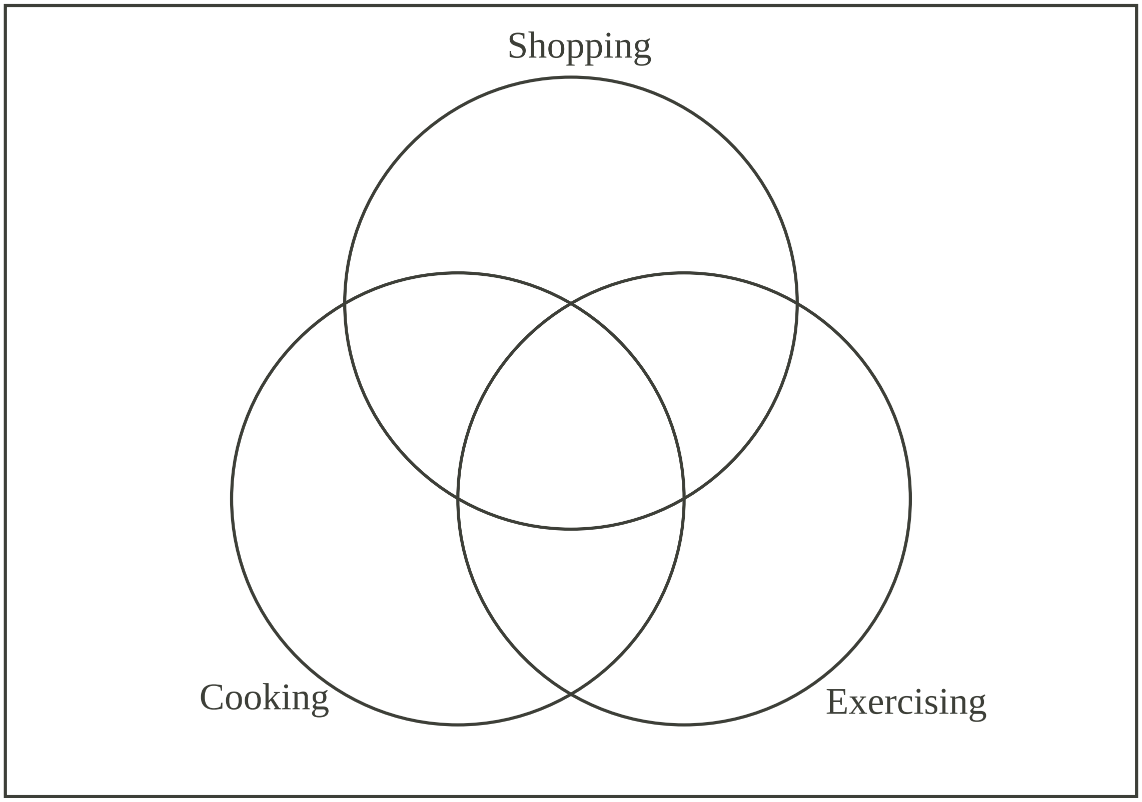
4. How many 4-digit odd numbers are there with no repeating digits?

5. Mark, Jane, Austin, Mike, Natalie, Laura, Steve and Nick are lining up for photos, but Mike and Jane don't want to be beside each other, and Laura wants to be beside Mark. How many ways can they line up?

6. [**Challenge**] How many different ways can you arrange letters of the word $MISSISSIPPI$?

7. [**Challenge**] You just bought 5 really cool paintings, but you only have space to put 3 of them up on your wall! How many ways can you put the paintings up on your wall assuming you don't care about the order that they're in?

Challenge Problem: A group of students was surveyed about their hobbies. With the help of the clues, complete the Venn diagram.



- Nobody dislikes all three hobbies
- 36 students like to go shopping
- 26 students like to exercise
- There are 44 students that like to shop or exercise
- There are twice as many students who only like to shop compared to the number of students who only like to cook
- Out of the students who don't like to exercise, half of that group likes to cook
- 16% of the whole group of students like all three hobbies
- Out of the students who like more than one hobby, two thirds of that group like shopping and exercising