Probability

Sets

Definition:
A set is simply a collection of objects. The objects can be anything, letters, numbers, people...
Sets are often labeled using upper case letters.

Example:
$A = \{1, 2, 3\}$. $A$ is the set containing the positive integers 1, 2, and 3. The order of the elements or members of the set does not matter.

Definitions:
A subset of a set is another set which contains all or some of the elements of a set, or nothing. The set with nothing in it is called the empty set or the null set, and is considered to be a subset of every set.

Example:
The subsets of the set $A$ given above are:

$\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{\} $

There is 1 subset of $A$ with no elements, 3 with one element, 3 with two elements, and 1 with three elements, a total of 8 subsets.

In general, if a set has $k$ elements in it, there are $2^k$ subsets as there are two choices for each element, put it in the subset or leave it out.
Definition:
The cardinality of a set $A$, denoted $n(A)$, is simply the number of elements in the set.

Example:
For the set $A$ given above, $n(A) = 3$.

Probability

Definitions:
The set of all possible outcomes for an activity, called an experiment, is called the sample space of the experiment.

Examples:
For tossing a coin, the sample space is $S = \{H, T\}$, denoting Heads and Tails.
For rolling a standard die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.
For rolling a pair of standard dice, the sample space consists of 36 ordered pairs of values, each of which come from the set $\{1, 2, 3, 4, 5, 6\}$. The sample space for rolling two dice is:

$$
\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}
$$

Definition:
An event is a subset of the sample space and the probability of any event, $E$, is defined to be

$$
p(E) = \frac{n(E)}{n(S)}.
$$

Example:
Consider the experiment of rolling a die. Define event $E$ to be rolling a 1, 3, or 5, ie $E = \{1, 3, 5\}$, and define event $F$ to be rolling a 1 or a 2, ie $F = \{1, 2\}$.

$$
p(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} \text{ and } p(F) = \frac{n(F)}{n(S)} = \frac{2}{6} = \frac{1}{3}.
$$
Definitions:
The event \((E \text{ or } F)\) is defined as either an outcome for \(E\) occurs or an outcome for \(F\) occurs. The subset which represents \((E \text{ or } F)\) is the \textit{union} of the sets representing \(E\) and \(F\).

The event \((E \text{ and } F)\) is defined an outcome for \(E\) occurs and an outcome for \(F\) occurs. The subset which represents \((E \text{ and } F)\) is the \textit{intersection} of the sets representing \(E\) and \(F\).

Example:
In the situation above where you are rolling a die, \(E \cup F = \{1, 2, 3, 5\}\) and \(E \cap F = \{1\}\).
Therefore \(p(E \cap F) = \frac{1}{6}\) and \(p(E \cup F) = \frac{4}{6} = \frac{2}{3}\).

Note that \(p(E \cup F) = p(E) + p(F) - p(E \cap F)\).
Problem Set

1. Let $A$ be the set of all prime numbers between 6 and 22? What are all subsets of $A$ with a cardinality of 2?

2. The corner store in your neighbourhood sells 5 different ice cream flavours. If you can get a banana split with any combination of any number of ice cream flavours, from no ice cream to all 5, how many different combinations could you make?

3. Let the set $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the set $D = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$. What is $C \cup D$? What is $C \cap D$?

4. You roll two standard dice and take the sum of the top numbers. What is the probability that this sum will be less than 7?

5. You are given a well-shuffled deck of 52 cards and asked to choose one card at random.

   (a) What is the probability that it will be a diamond?

   (b) What is the probability that it will be a face card (face cards are Jacks, Queens, Kings, and Aces)?

   (c) What is the probability that it will be both a diamond and a face card? What is the probability that it will be a diamond or a face card.

6. There are 40 balls in a hat, each of which are either red, blue, or green, and have a number from 1 to 5 written on them. If the probability of drawing a red ball with a “2” on it is $\frac{1}{10}$, the probability of drawing a ball that is either red or has a “2” on it or both is $\frac{1}{4}$, and there are 36 red balls in the hat, what is the probability of drawing a ball with a “2” on it?