The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Galois Contest

(Grade 10)

Wednesday, April 5, 2023
(in North America and South America)

Thursday, April 6, 2023
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

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Number of questions: 4 **Each question is worth 10 marks**

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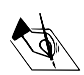
2. **FULL SOLUTION** parts indicated by 
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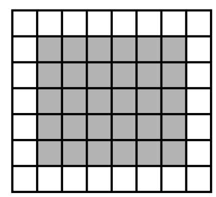
1. Jigsaw puzzles often have pieces that are arranged in a rectangular grid of rows and columns, where every cell in the grid represents one piece. The grid has two types of pieces: *edge pieces* which form the outer edge of the grid; and *middle pieces* which form the inside of the grid. In the example shown, there are 7 rows and 8 columns, and the middle pieces are shaded.



(a) How many pieces, in total, does a grid with 12 rows and 15 columns have?

(b) How many middle pieces does a grid with 6 rows and 4 columns have?

(c) If a grid has 14 middle pieces, then it either has s edge pieces or it has t edge pieces. Determine the values of s and t.

(d) A grid with 5 rows and c columns has the same number of edge pieces as middle pieces. Determine the value of c.

2. In an *Ing sequence*, the first term is a positive integer and each term after the first is determined in the following way:

- if a term, x, is odd, the next term is $x + 3$, and
- if a term, x, is even, the next term is $x + 4$.

For example, if the first term in an Ing sequence is 13, then the second term is 16, and the third term is 20.

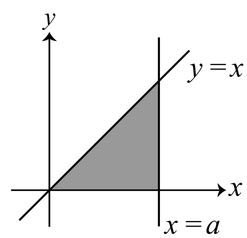
(a) If the first term in an Ing sequence is 7, what is the fifth term in the sequence?

(b) If the fifth term in an Ing sequence is 62, what are the two possibilities for the first term?

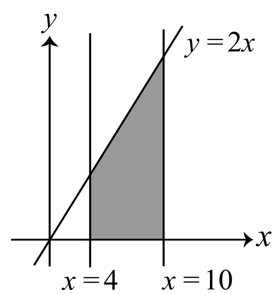
(c) If the first term in an Ing sequence is 49, determine the terms appearing in the sequence whose values are greater than 318 and less than 330.

(d) The number 18 appears somewhere in an Ing sequence after the first term. If the first term is the positive integer n, determine all possible values of n.

3. (a) The shaded triangle shown is bounded by the x-axis, the line $y = x$, and the line $x = a$, where $a > 0$. If the area of this triangle is 32, what is the value of a?



(b) A triangle is bounded by the x-axis, the line $y = 2x$, and the line $x = 10$. Diego draws the vertical line $x = 4$. This line divides the original triangle into a trapezoid, which is shaded, and a new unshaded triangle, as shown. What is the area of the shaded trapezoid?



(c) A triangle is bounded by the x-axis, the line $y = 3x$, and the line $x = 21$. Alicia draws the vertical line $x = c$, where $0 < c < 21$. This line divides the original triangle into a trapezoid and a new triangle. If the area of the trapezoid is 8 times the area of the new triangle, determine the value of c.

(d) A triangle is bounded by the x-axis, the line $y = 4x$, and the line $x = 1$. Ahmed draws his first vertical line at $x = p$, where $0 < p < 1$. This line divides the area of the original triangle in half. Ahmed then draws a second vertical line at $x = q$, where $0 < q < p$. This line divides the area of the triangle bounded by the x-axis, the line $y = 4x$, and the line $x = p$ in half. Ahmed continues this process of drawing vertical lines at decreasing values of x so that each such line divides the area of the previous triangle in half. If the 12th vertical line that he draws is at $x = k$, determine the value of k.

4. When people gather for a meeting, each person shakes hands with all, some or none of the other people, and never with the same person twice. When a handshake occurs between two people, this is counted as one handshake.

(a) At a meeting of 5 people, Amrita shook hands with exactly 1 person, Bin and Carlos each shook hands with exactly 2 people, Dennis shook hands with exactly 3 people, and Eloise did not shake hands with anyone. How many handshakes took place?

(b) At a meeting of 9 people, each participant said that they shook hands with exactly 3 people. Explain why this is not possible.

(c) At a meeting of 7 people, at least one handshake occurred within each group of 3 people. Determine the minimum possible number of handshakes, m, that took place at this meeting. A complete solution must include the value of m, an explanation of why the given conditions can be satisfied with some specific set of m handshakes, and an explanation of why fewer than m handshakes does not satisfy the given conditions.

2023 Galois Contest (English)

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(Grade 10)

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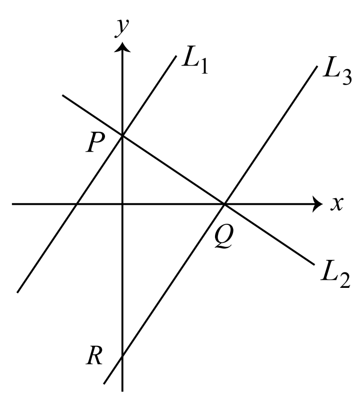
1. Alice and Bello contributed to the cost of starting a new business. The ratio of Alice's contribution to Bello's contribution was $3 : 8$.

   (a) If the cost of starting the new business was \$9240, what was Bello's contribution to this starting cost?

   (b) Alice and Bello divided up all profits in the first year of the business in the same ratio, $3 : 8$. Alice's share of the first year's total profit was \$1881. What was the total profit of the business for the first year?

   (c) In the second year, the business was changed so the share of that year's profits for Alice and Bello was in the ratio of $3 : (8 + x)$. If the profit for the second year was \$6400 and Bello's share of that profit was \$5440, determine the value of x.

2. In the diagram shown, line L_1 has equation $y = \frac{3}{2}x + k$, where $k > 0$, and L_1 intersects the y-axis at P. A second line, L_2, is drawn through P perpendicular to L_1, and intersects the x-axis at Q. A third line, L_3, is drawn through Q parallel to L_1, and intersects the y-axis at R.



   (a) What is the slope of L_2?

   (b) Written in terms of k, what is the x-coordinate of point Q?

   (c) If the area of $\triangle PQR$ is 351, determine the value of k.

3. The *prime factorization* of 324 is $2 \times 2 \times 3 \times 3 \times 3 \times 3$ or $2^2 \times 3^4$. Notice that 324 is a perfect square because it can be written in the form $(2 \times 3^2) \times (2 \times 3^2)$.
The prime factorization of 63 is $3^2 \times 7$. Notice that 63 is not a perfect square, but 63×7 is a perfect square, because $63 \times 7 = 3^2 \times 7^2 = (3 \times 7) \times (3 \times 7)$.

(a) The product $84 \times k$ is a perfect square. If k is a positive integer, what is the smallest possible value of k?

(b) The product $572 \times \ell$ is a perfect square. If ℓ is a positive integer less than 6000, what is the greatest possible value of ℓ?

(c) Show that if m is a positive integer less than 200, then $525\,000 \times m$ cannot be a perfect square.

(d) The list $10, 10^3, 10^5, \ldots, 10^{99}$ contains the fifty powers of 10 with odd integer exponents from 10^1 to 10^{99}, inclusive. Show that the sum of every choice of three different powers of 10 from this list is not a perfect square.

4. A *Bauman string* is a string of letters that satisfies the following two conditions.

- Each letter in the string is A, B, C, D, or E.
- No two adjacent letters in the string are the same.

For example, $AECD$ and $BDCEC$ are Bauman strings of length 4 and length 5, respectively, and $ABBC$ and $DAEEE$ are not Bauman strings.

(a) How many Bauman strings of length 5 are there in which the first letter and the last letter are both A?

(b) Determine the number of Bauman strings of length 6 that contain more than one B.

(c) Determine the number of Bauman strings of length 10 in which the first letter is C and the last letter is D.

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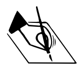
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
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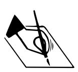
1. The operation $\triangle$ is defined by $a \triangle b = a(2b + 4)$ for integers a and b. For example, $3 \triangle 6 = 3(2 \times 6 + 4) = 3(16) = 48$.

    (a) What is the value of $5 \triangle 1$?

    (b) If $k \triangle 2 = 24$, what is the value of k?

    (c) Determine all values of p for which $p \triangle 3 = 3 \triangle p$.

    (d) Determine all values of m for which $m \triangle (m + 1) = 0$.

2. The organizer for a sports league with four teams has entered some of the end-of-season data into the table shown. Each team played 27 games and each game resulted in a win for one team and a loss for the other team, or in a tie for both teams. Each team earned 2 points for a win, 0 points for a loss, and 1 point for a tie.

Team Name	**Games Played**	**Number of Wins**	**Number of Losses**	**Number of Ties**	**Total Points**
P	27	10	14		23
Q	27				
R	27				25
S	27				

(a) How many ties did Team P have at the end of the season?

(b) Team Q had 2 more wins than Team P and 4 fewer losses than Team P. How many total points did Team Q have at the end of the season?

(c) Explain why Team R could not have finished the season with exactly 6 ties.

(d) At the end of the season, Team S had 4 more wins than losses. Show that Team S must have finished the season with a total of 31 points.

3. Rectangle $ABCD$ has vertices $A(0,0)$, $B(0,12)$, $C(6,12)$, and $D(6,0)$.

(a) Diagonals AC and BD intersect at point E. What is the area of $\triangle ADE$?

(b) Point $P(0,p)$ lies on line segment AB. The area of trapezoid $BCDP$ is twice the area of $\triangle PAD$. What is the value of p?

(c) The line passing through $U(0,u)$, $V(2,4)$ and $W(6,w)$ divides $ABCD$ into two trapezoids. Determine all possible pairs of points U and W for which the ratio of the areas of these two trapezoids is $5:3$.

4. (a) If $\dfrac{5}{x} + \dfrac{14}{y} = 2$ and $x = 6$, what is the value of y?

(b) Determine all possible ordered pairs of positive integers (x,y) that are solutions to the equation $\dfrac{4}{x} + \dfrac{5}{y} = 1$.

(c) Consider the equation $\dfrac{16}{x} + \dfrac{25}{y} = p$, where p is a prime number and $p \geq 5$. Determine all possible values of p for which there is at least one ordered pair of positive integers (x,y) that is a solution to the equation.

2021 Galois Contest (English)

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1. The letters A and B are used to create a pattern consisting of a number of rows. The pattern starts with a single A. The rows alternate between A's and B's, and the number of letters in each row is twice the number of letters in the previous row. The first 4 rows of the pattern are shown.

Row 1	A
Row 2	BB
Row 3	$AAAA$
Row 4	$BBBBBBBB$

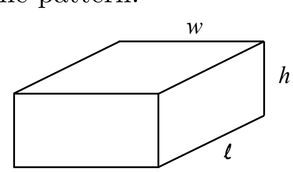
(a) If the pattern consists of 6 rows, how many letters are in the 6^{th} row of the pattern?

(b) If the pattern consists of 6 rows, what is the total number of letters in the pattern?

(c) If the total number of letters in the pattern is 63, determine the number of A's in the pattern and the number of B's in the pattern.

(d) If the total number of letters in the pattern is 4095, determine the difference between the number of A's and the number of B's in the pattern.

2. For a rectangular prism with length ℓ, width w, and height h as shown, the surface area is given by the formula $A = 2\ell w + 2\ell h + 2wh$ and the volume is given by the formula $V = \ell wh$.



(a) What is the surface area of a rectangular prism with length 2 cm, width 5 cm, and height 9 cm?

(b) A rectangular prism with height 10 cm has a square base. The volume of the prism is 160 cm^3. What is the side length of the square base?

(c) A rectangular prism has a square base with area 36 cm^2. The surface area of the prism is 240 cm^2. Determine the volume of the prism.

(d) A rectangular prism has length k cm, width $2k$ cm, and height $3k$ cm, where $k > 0$. The volume of the prism is x cm^3. The surface area of the prism is x cm^2. Determine the value of k.

3. Jodi multiplied the numbers 2 and 5 to get a product of 10. She added 4 to each of her original numbers to get 6 and 9. She multiplied these new numbers to get a product of 54. Jodi noticed that each of the digits in the new product, 54, was 4 more than the corresponding digits in the first product, 10.

$$\begin{array}{ccccccc} 2 & \times & 5 & = & 1 & 0 & \\ {\scriptstyle +4}\downarrow & & {\scriptstyle +4}\downarrow & & {\scriptstyle +4}\downarrow & \downarrow{\scriptstyle +4} & \\ 6 & \times & 9 & = & 5 & 4 & \end{array}$$

The pair $(2,5)$ is an example of a *RadPair*.
In general, a pair of positive integers (a,b) with $a \le b \le 9$ and for which the product ab is a two-digit integer is called a *RadPair* if there exists a positive integer d such that

- the product $(a+d)(b+d)$ is a two-digit integer, and
- the ones (units) digit of the product $(a+d)(b+d)$ equals d plus the ones digit of the product ab, and
- the tens digit of the product $(a+d)(b+d)$ equals d plus the tens digit of the product ab.

(a) Show that $(2,8)$ is a RadPair.

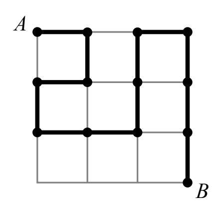
(b) Show that $(3,6)$ is not a RadPair.

(c) For which positive integers x with $x \le 6$ is $(x,6)$ a RadPair?

(d) Determine, with justification, the number of RadPairs (a,b) with $a \le b$.

4. In an $n \times n$ grid of unit squares, each point at which two grid lines meet is called a *vertex*, and so there are $(n+1)^2$ *vertices*. The top left corner vertex is labeled A and the bottom right corner vertex is labeled B. A *path* from A to B is a sequence of unit edges that

- each connect two adjacent vertices, and
- when connected, form a sequence of vertices that begins at A, ends at B, and
- passes through each vertex at most once.



The length of such a path is the number of unit edges in the path. For example, in a 3×3 grid, a path of length 12 between A and B is shown.

(a) In a 2×2 grid, determine the number of paths of any length from A to B.

(b) Explain why there cannot be a path from A to B of odd length in a 10×10 grid.

(c) In a 4×4 grid, determine the number of paths of length 10 from A to B.

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Galois Contest

(Grade 10)

Wednesday, April 10, 2019
(in North America and South America)

Thursday, April 11, 2019
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

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Number of questions: 4 **Each question is worth 10 marks**

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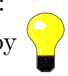
1. **SHORT ANSWER** parts indicated by 
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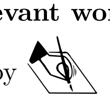
2. **FULL SOLUTION** parts indicated by 
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7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. The Galois Restaurant is in a region that adds 10% sales tax onto the price of food and drinks purchased at a restaurant. The prices listed on their menu do not include the sales tax.

   (a) From the menu, Becky orders a plate of lasagna listed for \$7.50, a side salad listed for \$5.00, and a lemonade listed for \$3.00. After tax is included, how much is Becky's total bill?

   (b) A burrito is listed on the menu for \$6.00. After tax is included, what is the greatest number of burritos that Jackson can buy if he has \$50.00?

   (c) On the Galois Restaurant menu, hotdogs are listed at the regular price of \$5.00. The restaurant has the following promotional deals:

   - On Mondays, if you buy a hotdog at the regular menu price of \$5.00, then the price for a second hotdog is \$4.50.
   - On Tuesdays, you pay half the tax on all hotdogs.

   Chase bought two hotdogs on Monday and then two hotdogs on Tuesday. After tax is included, determine on which day Chase spent less money.

2. The hypotenuse of right-angled $\triangle AOB$ lies on the line with equation $y = -2x + 12$, as shown in Figure 1. The legs of $\triangle AOB$ lie on the axes.

(a) What is the area of $\triangle AOB$?

(b) A second line passes through O and is perpendicular to the first line, as shown in Figure 2. The two lines intersect at C. Determine the coordinates of C.

(c) The second line passes through the point D in the first quadrant, as shown in Figure 3. Points E and F are positioned on the axes so that $DEOF$ is a rectangle. If the area of $DEOF$ is 1352, determine the coordinates of D.

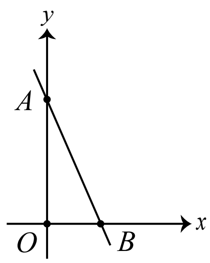

Figure 1

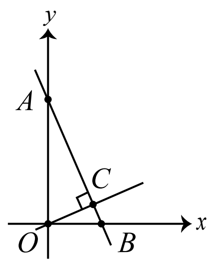

Figure 2

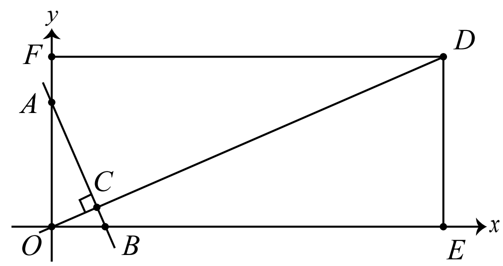

Figure 3

3. If n is a positive integer, the notation $n!$ (read "n factorial") is used to represent the product of the integers from 1 to n. That is, $n! = n(n-1)(n-2)\cdots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$.

(a) What is the largest positive integer m for which 2^m is a divisor of $9!$?

(b) What is the smallest value of n for which $n!$ is divisible by 7^2?

(c) Explain why there is no positive integer n for which $n!$ is divisible by 7^7 but is not divisible by 7^8.

(d) Show that there is exactly one positive integer n for which

$$n! = 2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19 \cdot 23, \text{ and}$$

$$a + b + c + d = 45$$

for some positive integers a, b, c, d.

4. A positive integer is *digit-balanced* if each digit d, with $0 \leq d \leq 9$, appears at most d times in the integer. For example, 13224 is digit-balanced, but 21232 is not.

(a) Explain why a digit-balanced integer is not divisible by 10.

(b) How many 4-digit integers have all non-zero digits and are *not* digit-balanced?

(c) Determine all positive integers k for which there exist digit-balanced positive integers m and n, where $m + n = 10^k$ and m and n each have k digits.

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(Grade 10)

Thursday, April 12, 2018
(in North America and South America)

Friday, April 13, 2018
(outside of North America and South America)

UNIVERSITY OF WATERLOO

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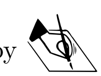
2. **FULL SOLUTION** parts indicated by 
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1. (a) Given that $x \neq 0$, simplify the expression $\dfrac{12x^2}{3x}$.

   (b) What is the value of the expression $\dfrac{12x^2}{3x}$ when $x = 5$?

   (c) Given that $n = 2m$ and $m \neq 0$, what is the value of the expression $\dfrac{8mn}{3m^2}$?

   (d) If $q = 6$, determine all positive integers p for which $3 \leq \dfrac{8p^2q}{5pq^2} \leq 4$.

2. Here are two facts about circles:

   - If points A, B, C lie on a circle so that $\angle ABC = 90^\circ$, then AC is a diameter of the circle. This means that in Figure 1, AC is a diameter of the circle.
   - If points D, E, F lie on a circle so that EF is a diameter, then $\angle EDF = 90^\circ$. This means that in Figure 2, $\angle EDF = 90^\circ$.

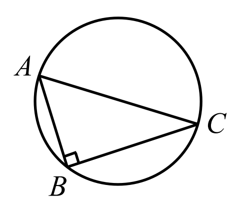

Figure 1

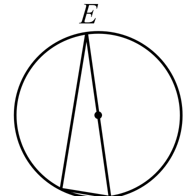

Figure 2

   (a) In Figure 1 above, $AB = 8$ and $BC = 15$. What is the length of diameter AC?

   (b) In Figure 2 above, $DE = 24$ and the radius of the circle is 13. What is the length of DF?

   (c) In Figure 3, points P, Q, R, and S are on a circle with centre O. Also, SQ is a diameter of the circle and O is joined to R. If $SP = PQ$ and $\angle RQP = 80^\circ$, determine the measure of $\angle ROQ$ and the measure of $\angle RSQ$.

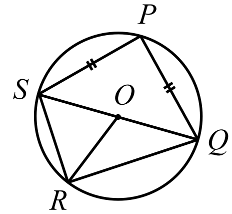

Figure 3

3. Cylinder A has radius 12 and height 25. Cylinder B has radius 9 and height h. Cylinder A is filled with water to a depth of 19. Cylinder B is empty. Cylinder B is lowered to the bottom of Cylinder A, as shown. Depending on the value of h,

(i) some water may spill out of Cylinder A onto the ground (Figure 1), or

(ii) some water may pour into Cylinder B (Figure 2), or

(iii) (i) then (ii).

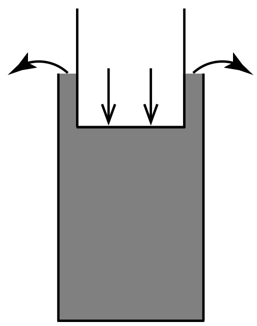

Figure 1

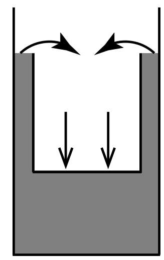

Figure 2

The walls and bases of the two cylinders are thin enough that their width can be ignored.

(a) Suppose that $h = 30$. What is the volume of water that spills out of Cylinder A onto the ground?

(b) Suppose that $h = 20$. Determine the volume of water that spills out of Cylinder A onto the ground and the depth of water in Cylinder B when it is on the bottom of Cylinder A.

(c) Determine the range of values of h so that when Cylinder B is on the bottom of Cylinder A, there is some water in Cylinder B but it is not full.

4. For each positive integer k, we define $C(k)$ to be the number of ways in which k can be written as the sum of one or more consecutive positive integers. For example, $C(21) = 4$ because 21 can be written as

$$21, \quad 10 + 11, \quad 6 + 7 + 8, \quad \text{and} \quad 1 + 2 + 3 + 4 + 5 + 6,$$

and there are no other lists of one or more consecutive positive integers whose sum is 21.

(a) What is the value of $C(45)$?

(b) The positive integer m equals the sum of the positive integers from 4 to n, inclusive. Determine the values of a and b, with $a < b$, for which $m = \frac{1}{2}(n + a)(n + b)$ for each positive integer $n \geq 4$.

(c) Determine the value of $C(2 \times 3^4 \times 5^6)$.

(d) Determine the smallest positive integer k for which $C(k) = 215$.

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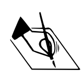
2. **FULL SOLUTION** parts indicated by 
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1. On Monday, Daniel had 90 cups, each of which was either purple or yellow. He distributed the cups among three boxes as follows:

   Box D: 9 purple and 23 yellow cups for a total of 32 cups
   Box E: 6 purple and 24 yellow cups for a total of 30 cups
   Box F: 28 cups in total

   (a) What percentage of the cups in Box E were purple?

   (b) Of the 90 cups that Daniel had on Monday, 30% were purple. How many of the cups in Box F were purple?

   (c) On Tuesday, Avril brought 9 more purple cups and included them with Daniel's cups. Barry brought some yellow cups and included them with Daniel's cups and Avril's cups. The percentage of cups that were purple was again 30%. How many cups did Barry bring?

2. The Breakfast Restaurant has a special pricing day. If a customer arrives at the restaurant between 4:30 a.m. and 7:00 a.m., the time that they arrive in hours and minutes becomes the price that they pay in dollars and cents. For example, if a customer arrives at 5:23 a.m., they will pay \$5.23.

   (a) Abdi arrived at 5:02 a.m. and Caleigh arrived at 5:10 a.m. In total, how much did they pay?

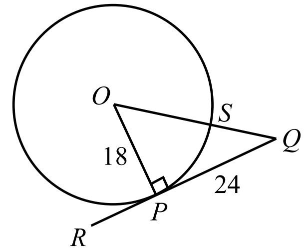
   (b) Robert arrived 10 minutes before Emily, and both arrived during the period of the special pricing. In total, they paid \$12.34. What were their arrival times?

   (c) Isaac and Jacob arrived together and Karla arrived after. All three arrived during the period of the special pricing. In total, they paid \$18.55. What was the minimum amount that Karla could have paid?

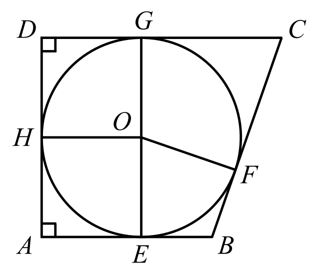
   (d) Larry and Mio arrived separately during the period of the special pricing. In total, they paid \$11.98. Determine the ranges of times during which Larry could have arrived.

3. A *tangent* to a circle is a line or line segment that touches the circle in exactly one place and would not touch the circle again, even if extended infinitely in both directions. When a tangent to a circle with centre O touches the circle at P, radius OP is perpendicular to the tangent.

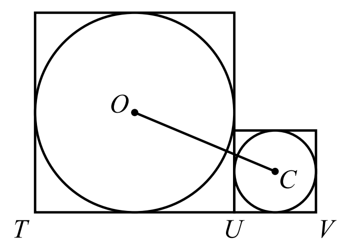
(a) In the diagram, O is the centre of the circle with radius 18. QR is tangent to the circle at P. Line segment OQ intersects the circle at S. Determine the length of SQ.



(b) A circle is said to be *inscribed* in a quadrilateral if each side of the quadrilateral is tangent to the circle. A circle with centre O is inscribed in quadrilateral $ABCD$, touching AB at E, BC at F, CD at G, and DA at H, as shown. If the radius of the circle is 12, $OB = 15$, $OC = 20$, and $\angle BAD = \angle ADC = 90°$, what is the perimeter of quadrilateral $ABCD$?

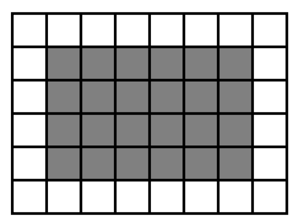


(c) Circles with centres O and C are inscribed in squares, as shown. The area of the larger square is 289 and the area of the smaller square is 49. If T, U and V lie on a straight line, determine the length of OC.



4. A *Koeller-rectangle*:

- is an m by n rectangle where m, n are integers with $m \geq 3$ and $n \geq 3$,
- has lines drawn parallel to its sides to divide it into 1 by 1 squares, and
- has the 1 by 1 squares along its sides unshaded and the 1 by 1 squares that do not touch its sides shaded.



An example of a Koeller-rectangle with $m = 8$ and $n = 6$ is shown.

For a given Koeller-rectangle, let r be the ratio of the shaded area to the unshaded area.

(a) Determine the value of r for a Koeller-rectangle with $m = 14$ and $n = 10$.

(b) Determine all possible positive integer values of u for which there exists a Koeller-rectangle with $n = 4$ and $r = \dfrac{u}{77}$.

(c) Determine all prime numbers p for which there are exactly 17 positive integer values of u for Koeller-rectangles with $n = 10$ and $r = \dfrac{u}{p^2}$.

2017 Galois Contest (English)

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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Galois Contest

(Grade 10)

Wednesday, April 13, 2016
(in North America and South America)

Thursday, April 14, 2016
(outside of North America and South America)

UNIVERSITY OF WATERLOO

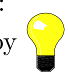
Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
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WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

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7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

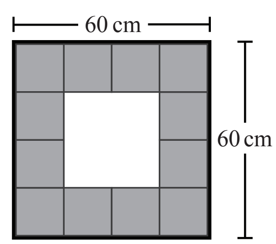
1. Liza has a row of buckets. The first bucket contains 17 green discs and 7 red discs. Each bucket after the first contains 1 more green disc and 3 more red discs than the previous bucket.

   (a) Which bucket contains 16 red discs?

   (b) In which bucket is the number of red discs equal to the number of green discs?

   (c) There is a bucket in which the number of red discs is twice the number of green discs. In total, how many discs are in this bucket?

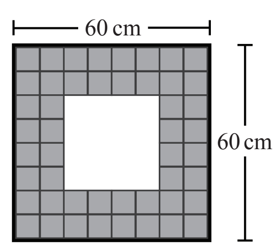
2. Judy has square plates, each with side length 60 cm. A plate is *Shanks-Decorated* if identical shaded squares are drawn along the outer edges of the plate, as shown. The diagram shows an example of a plate that is Shanks-Decorated with 12 shaded squares.



   (a) Judy's first plate is Shanks-Decorated with 36 shaded squares. What is the side length of each shaded square?

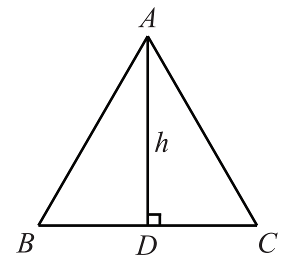
   (b) When a second plate is Shanks-Decorated, an area of 1600 cm^2 is left unshaded in the centre of the plate. What is the side length of each shaded square?

   (c) A plate is *Double-Shanks-Decorated* if two layers of identical shaded squares are drawn along the outer edges of the plate, as shown. The diagram shows an example of a plate that is Double-Shanks-Decorated with 48 shaded squares.

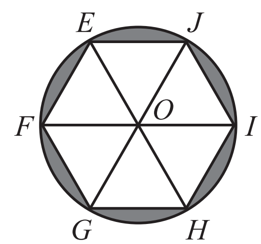


   A new plate is Double-Shanks-Decorated and an area of 2500 cm^2 is left unshaded in the centre of the plate. Determine the number of shaded squares.

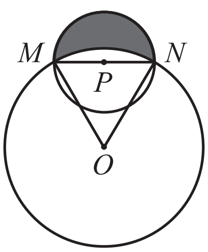
3. (a) In the diagram, $\triangle ABC$ is equilateral with side length 6 and D is the midpoint of BC. Determine the *exact* value of h, the height of $\triangle ABC$.



(b) In the diagram, a circle with centre O has radius 6. Regular hexagon $EFGHIJ$ has sides of length 6 and vertices on the circle. Determine the *exact* area of the shaded region.



(c) A circle has centre O and radius r. A second circle has centre P and diameter MN. The circles intersect at M and N. If $MN = \ r$, determine the *exact* area of the shaded region, in terms of r.



4. The prime factorization of 45 is $3^2 5^1$. In general, the prime factorization of an integer $n \geq 2$ is of the form $p_1^{a_1} p_2^{a_2} p_3^{a_3} \dots p_k^{a_k}$ where $p_1, p_2, \dots, p_k$ are different prime numbers and $a_1, a_2, \dots, a_k$ are positive integers.
Given an input of an integer $n \geq 2$, the *Barbeau Process* outputs the number equal to $n\left(\frac{a_1}{p_1} + \frac{a_2}{p_2} + \frac{a_3}{p_3} + \cdots + \frac{a_k}{p_k}\right)$.
For example, given an input of 45, the Barbeau Process outputs $45\left(\frac{2}{3} + \frac{1}{5}\right) = 30 + 9 = 39$, since the prime factorization of 45 is $3^2 5^1$.

(a) Given an input of 126, what number does the Barbeau Process output?

(b) Determine all pairs (p, q) of different prime numbers such that the Barbeau Process with input $p^2 q$ outputs 135.

(c) Determine all triples (a, b, c) of positive integers such that the Barbeau Process with input $2^a 3^b 5^c$ outputs $4 \times 2^a 3^b 5^c$.

(d) Determine all integer values of n with $2 \leq n < 10^{10}$ such that the Barbeau Process with input n outputs $3n$.

2016 Galois Contest (English)

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Galois Contest

(Grade 10)

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(in North America and South America)

Friday, April 17, 2015
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

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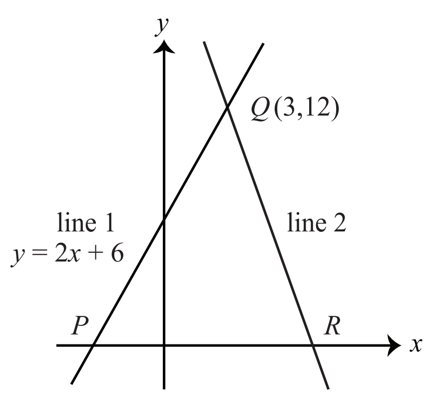
1. (a) In the diagram, line 1 has equation $y = 2x+6$ and crosses the x-axis at P. What is the x-intercept of line 1?

   (b) Line 2 has slope -3 and intersects line 1 at $Q(3, 12)$, as shown. Determine the equation of line 2.

   (c) Line 2 crosses the x-axis at R, as shown. Determine the area of $\triangle PQR$.



2. On Wednesday, students at six different schools were asked whether or not they received a ride to school that day.

   (a) At School A, there were 330 students who received a ride and 420 who did not. What percentage of the students at School A received a ride?

   (b) School B has 240 students, of whom 30% received a ride. How many more of the 240 students in School B needed to receive a ride so that 50% of the students in School B got a ride?

   (c) School C has 200 students, of whom 45% received a ride. School D has 300 students. When School C and School D are combined, the resulting group has 57.6% of students who received a ride. If x% of the students at School D received a ride, determine x.

   (d) School E has 200 students, of whom n% received a ride. School F has 250 students, of whom $2n$% received a ride. When School E and School F are combined, between 55% and 60% of the resulting group received a ride. If n is a positive integer, determine all possible values of n.

3. (a) If $n + 5$ is an even integer, state whether the integer n is even or odd.

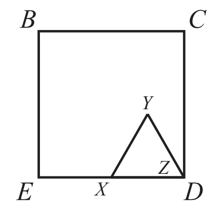
(b) If c and d are integers, explain why $cd(c + d)$ is always an even integer.

(c) Determine the number of ordered pairs (e, f) of positive integers where

- $e < f$,
- $e + f$ is odd, and
- $ef = 300$.

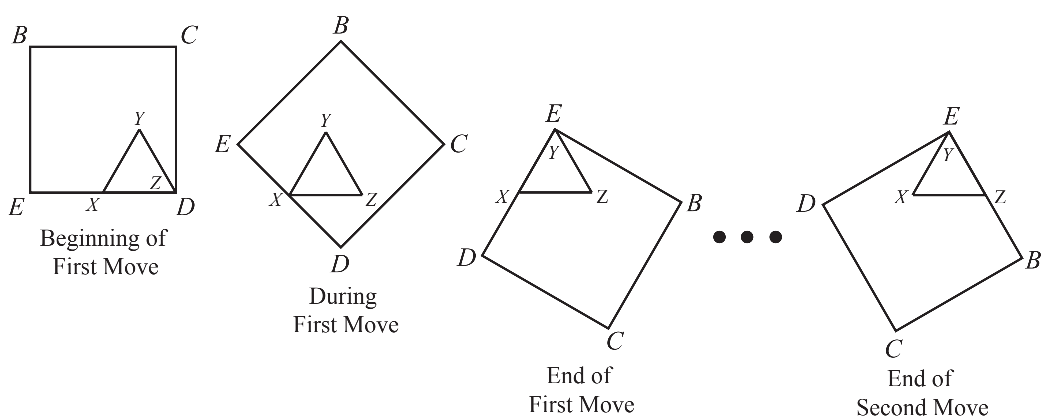
(d) Determine the number of ordered pairs (m, n) of positive integers such that $(m + 1)(2n + m) = 9000$.

4. In the diagram, square $BCDE$ has side length 2. Equilateral $\triangle XYZ$ has side length 1. Vertex Z coincides with D and vertex X is on ED.



(a) What is the measure of $\angle YXE$?

(b) A move consists of rotating the square clockwise around a vertex of the triangle until a side of the square first meets a side of the triangle. The first move is a rotation about X and the second move is a rotation about Y, as shown in the diagrams. (Note that the vertex of the triangle about which the square rotates remains in contact with the square during the rotation.)



In subsequent moves, the square rotates about vertex Z, then X, then Y, and so on. Determine, with justification, the total number of moves made from the beginning of the first move to when vertex D next coincides with a vertex of the triangle.

(c) Determine the length of the path travelled by point E from the beginning of the first move to when square $BCDE$ first returns to its original position (that is, when D next coincides with Z and XZ lies along ED).

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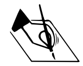
	4.	For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
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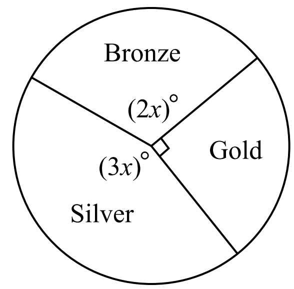
1. The pie chart shows the distribution of the number of bronze, silver and gold medals in a school's trophy case.



 (a) What is the value of x?

 (b) Write the ratio of the number of bronze medals to the number of silver medals to the number of gold medals in lowest terms.

 (c) If there is a total of 80 medals in the trophy case, determine the number of bronze medals, the number of silver medals, and the number of gold medals in the trophy case.

 (d) The trophy case begins with the same number of each type of medal as in part (c). A teacher then finds a box with medals and adds them to the trophy case. The ratio of the number of bronze medals, to the number of silver medals, to the number of gold medals is unchanged. What is the smallest number of medals that could now be in the trophy case?

2. An airplane holds a maximum of 245 passengers. To accommodate the extra expense of transporting luggage, passengers are charged a baggage fee of \$20 for the first bag checked plus \$7 for each additional bag checked. (Passengers who do not check a bag are not charged a baggage fee.)

 (a) On one flight, 200 passengers checked exactly one bag and the other 45 passengers checked exactly two bags. Determine the total of the baggage fees for checked bags.

 (b) On a second flight, the plane was again completely full. Every passenger checked exactly one or two bags. If a total of \$5173 in baggage fees were collected, how many passengers checked exactly two bags?

 (c) On a third flight, exactly \$6825 was collected in baggage fees. Explain why there must be at least one passenger who checked at least three bags.

 (d) On a fourth flight, exactly \$142 was collected in baggage fees. Explain why there must be at least one passenger who checked at least three bags.

3. Cards in a deck are numbered consecutively with positive integers. The cards are selected in pairs, (a, b) with $a < b$, to create a given sum, $a + b$. For example, Anna has a set of cards numbered from 1 to 50 and she is required to create a sum of 60. Two of the pairs that she can select are $(10, 50)$ and $(25, 35)$.

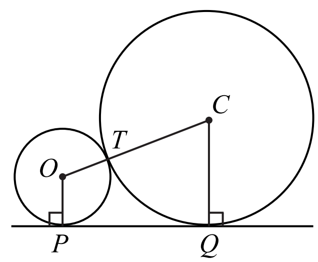
(a) Emily has a set of 10 cards numbered consecutively from 1 to 10. There are exactly 3 pairs that she can select, each having a sum of 8. List the 3 pairs.

(b) Silas has a set of 10 cards numbered consecutively from 1 to 10. Determine the number of pairs that he can select with a sum of 13.

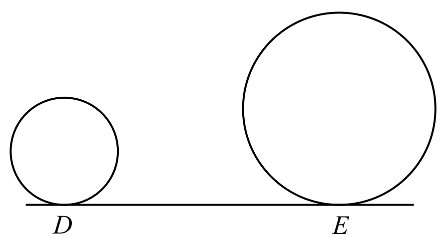
(c) Daniel has a set of k cards numbered consecutively from 1 to k. He can select exactly 10 pairs that have a sum of 100. What is the value of k?

(d) Derrick has a set of 75 cards numbered consecutively from 1 to 75. He can select exactly 33 pairs that have a sum of S. Determine, with justification, all possible values of S.

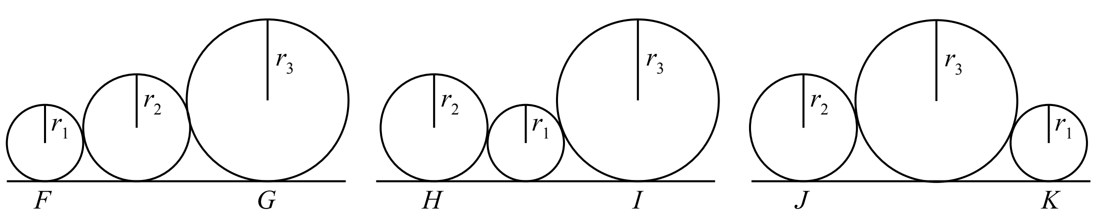
4. (a) A circle of radius 2 and a circle of radius 5 are externally tangent to each other at T and tangent to a horizontal line at points P and Q, as shown. If points O and C are the centres of the circles, then O, T, C are collinear and both OP and CQ are perpendicular to PQ. By constructing a line segment passing through O and parallel to PQ, determine the distance between P and Q.



(b) A circle of radius 4 and a circle of radius 9 are tangent to a horizontal line at points D and E, as shown. A third circle can be placed between these two circles so that it is externally tangent to each circle and tangent to the horizontal line. If $DE = 24$, determine with justification, the radius of this third circle.



(c) Three circles with radii $r_1 < r_2 < r_3$ are placed so that they are tangent to a horizontal line, and so that adjacent circles are externally tangent to each other. F, G, H, I, J, and K are the points of tangency of the circles to the horizontal line, as shown. The lengths of FG, HI, JK, in no particular order, are 18, 20 and 22. Determine, with justification, the values of r_1, r_2 and r_3.



2014 Galois Contest (English)

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Galois Contest

(Grade 10)

Thursday, April 18, 2013
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Friday, April 19, 2013
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Calculators are permitted — **Each question is worth 10 marks**

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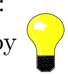
1. **SHORT ANSWER** parts indicated by 
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1. (a) Find an equation of the line that passes through the points $(2, 0)$ and $(0, 4)$.

   (b) Rewrite the equation of the line from part (a) in the form $\frac{x}{c} + \frac{y}{d} = 1$, where c and d are integers.

   (c) State the x-intercept and the y-intercept of the line $\frac{x}{3} + \frac{y}{10} = 1$.

   (d) Determine the equation of the line that passes through the points $(8, 0)$ and $(2, 3)$ written in the form $\frac{x}{e} + \frac{y}{f} = 1$, where e and f are integers.

2. A thick red candle and a thin green candle are each 100 cm tall. These two candles are lit at the same time. As the candles burn, their heights decrease at constant but different rates. The red candle takes 600 minutes to burn completely. The green candle takes 480 minutes to burn completely.

   (a) By how much will the height of the red candle have decreased 180 minutes after being lit?

   (b) How many minutes after being lit will the green candle be 80 cm tall?

   (c) How much taller will the red candle be than the green candle 60 minutes after they are lit?

   (d) How many minutes after being lit will the red candle be 7 cm taller than the green candle?

3. The even positive integers are listed in order and arranged into rows, as shown, and described below.

Row Number				
1	2			
2	4	6		
3	8	10	12	
4	14	16	18	20
		$\vdots$		

Each new row includes one more integer than the previous row. The last number in each row is the product of the row number and the next largest integer. For example, the last number in the 4^{th} row is 4×5. You may use this fact without proving it.

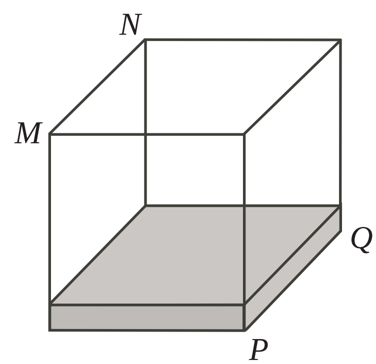
(a) List the numbers in the 7th row of the table.

(b) What are the first and last numbers in the 100th row of the table?

(c) The last number in row r is L. The first number in row $(r+2)$ is F. Determine the smallest possible value for r such that $F + L$ is at least 2013.

4. A cube with edge length 9 cm contains a certain amount of water.

(a) When the cube has one face resting on the ground, the depth of the water is 1 cm, as shown. What is the volume of water in the cube?



(b) The cube is turned so that one edge, PQ, is on the ground with the opposite edge, MN, directly above it. Calculate the depth of the water in the cube.

(c) The cube is now positioned so that a single corner, P, is on the ground with the opposite corner, N, directly above it. To the nearest hundredth of a centimetre, calculate the depth of the water in the cube.

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Galois Contest

(Grade 10)

Thursday, April 12, 2012
(in North America and South America)

Friday, April 13, 2012
(outside of North America and South America)

STRONGER COMMUNITIES TOGETHER™

Do not open this booklet until instructed to do so.

Time: 75 minutes **Number of questions:** 4
Calculators are permitted **Each question is worth 10 marks**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

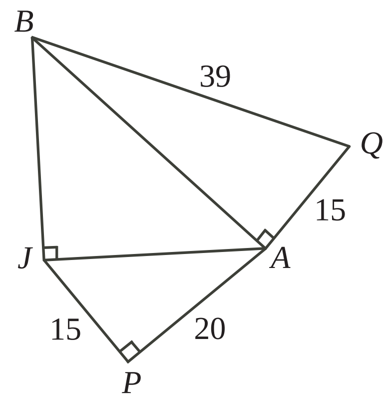
- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published in the FGH Results on our Web site, http://www.cemc.uwaterloo.ca.

TIPS:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write all answers in the answer booklet provided.
	3.	For questions marked, place your answer in the appropriate box in the answer booklet and **show your work**.
	4.	For questions marked, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
	5.	Diagrams are *not* drawn to scale. They are intended as aids only.

1. Adam and Budan are playing a game of Bocce. Each wants their ball to land closest to the jack ball. The positions of Adam's ball, A, Budan's ball, B, and the jack ball, J, are shown in the diagram.



   (a) What is the distance from A to J?

   (b) What is the distance from B to A?

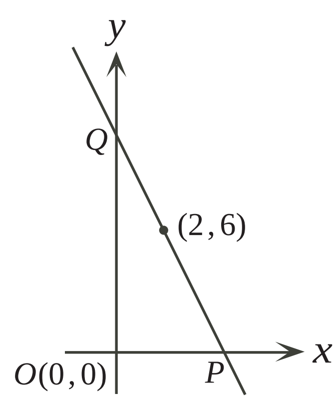
   (c) Determine whose ball is closer to the jack ball, Adam's or Budan's.

2. (a) When the numbers $25, 5$ and 29 are taken in pairs and averaged, what are the three averages?

   (b) When the numbers $2, 6$ and n are taken in pairs and averaged, the averages are 11, 4 and 13. Determine the value of n.

   (c) There are three numbers a, b and 2. Each number is added to the average of the other two numbers. The results are $14, 17$ and 21. If $2 < a < b$, determine the values of a and b.

3. The diagram shows one of the infinitely many lines that pass through the point $(2, 6)$.

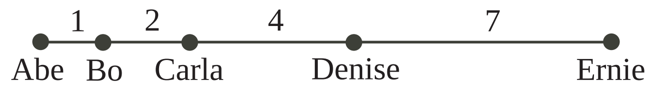


   (a) A line through the point $(2, 6)$ has slope -3. Determine the x- and y-intercepts of this line.

   (b) Another line through the point $(2, 6)$ has slope m. Determine the x- and y-intercepts of this line in terms of m.

   (c) A line through the point $(2, 6)$ has slope m, and crosses the positive x-axis at P and the positive y-axis at Q, as shown. Determine the two values of m for which $\triangle POQ$ has an area of 25.

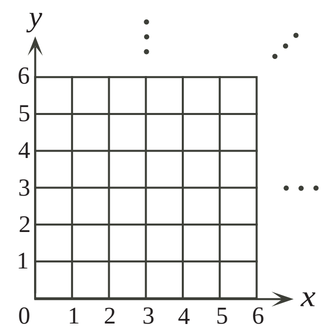
4. (a) In Town A, five students are standing at different intersections on the same east-west street, as shown. The distances between adjacent intersections are given in kilometres.



The students agree to meet somewhere on the street such that the total distance travelled by all five students is as small as possible. Where should the students meet?

(b) In Town B, there is an even number of students. The students are standing at different intersections on a straight north-south street. The students agree to meet somewhere on the street that will make the total distance travelled as small as possible. With justification, determine all possible locations where the students could meet.

(c) In Town C, the streets run north-south and east-west forming a positive xy-plane with intersections every 1 km apart, as shown. One hundred students are standing at different intersections. The first 50 students, numbered 1 to 50, stand so that the student numbered k stands at intersection $(2^k, k)$. (For example, student 5 stands at $(32, 5)$.) The remaining students, numbered 51 to 100, stand so that the student numbered j stands at intersection $(j - 50, 2j - 100)$. The students can only travel along the streets, and they agree to meet at an intersection that will make the total distance travelled by all students as small as possible. With justification, determine all possible intersections at which the students could meet.



2012 Galois Contest (English)

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2011 Galois Contest (Grade 10)

Wednesday, April 13, 2011

1. Jackson gave the following rule to create sequences:

   "If x is a term in your sequence, then the next term in your sequence is $\dfrac{1}{1-x}$."

   For example, Mary starts her sequence with the number 3.

   The second term of her sequence is $\dfrac{1}{1-3} = \dfrac{1}{-2} = -\dfrac{1}{2}$. Her sequence is now $3, -\frac{1}{2}$.

   The third term of her sequence is $\dfrac{1}{1-(-\frac{1}{2})} = \dfrac{1}{\frac{3}{2}} = \dfrac{2}{3}$. Her sequence is now $3, -\frac{1}{2}, \frac{2}{3}$.

   The fourth term of her sequence is $\dfrac{1}{1-\frac{2}{3}} = \dfrac{1}{\frac{1}{3}} = 3$. Her sequence is now $3, -\frac{1}{2}, \frac{2}{3}, 3$.

   Fabien starts his sequence with the number 2, and continues using Jackson's rule until the sequence has 2011 terms.

   (a) What is the second term of his sequence?

   (b) What is the fifth term of his sequence?

   (c) How many of the 2011 terms in Fabien's sequence are equal to 2? Explain.

   (d) Determine the sum of all of the terms in his sequence.

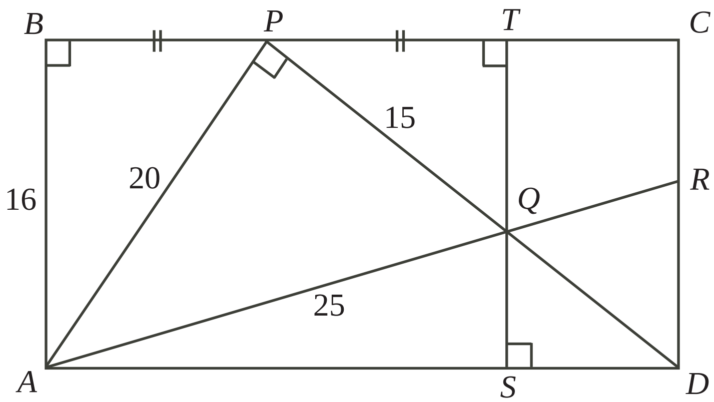
2. Alia has a bucket of coins. Each coin has a zero on one side and an integer greater than 0 on the other side. She randomly draws three coins, tosses them and calculates a score by adding the three numbers that appear.

   (a) On Monday, Alia draws coins with a 7, a 5 and a 10. When she tosses them, they show $7, 0$ and 10 for a score of 17. What other scores could she obtain by tossing these same three coins?

   (b) On Tuesday, Alia draws three coins and tosses them three times, obtaining scores of $60, 110$ and 130. On each of these tosses, exactly one of the coins shows a 0. Determine the maximum possible score that can be obtained by tossing these three coins.

   (c) On Wednesday, Alia draws one coin with a 25, one with a 50, and a third coin. She tosses these three coins and obtains a score of 170. Determine all possible numbers, other than zero, that could be on the third coin.

3. In rectangle $ABCD$, P is a point on BC so that $\angle APD = 90^\circ$. TS is perpendicular to BC with $BP = PT$, as shown. PD intersects TS at Q. Point R is on CD such that RA passes through Q. In $\triangle PQA$, $PA = 20$, $AQ = 25$ and $QP = 15$.



   (a) Determine the lengths of BP and QT.

   (b) Show that $\triangle PQT$ and $\triangle DQS$ are similar. That is, show that the corresponding angles in these two triangles are equal.

   (c) Determine the lengths of QS and SD.

   (d) Show that $QR = RD$.

4. For a positive integer n, the n^{th} triangular number is $T(n) = \dfrac{n(n+1)}{2}$.
   For example, $T(3) = \frac{3(3+1)}{2} = \frac{3(4)}{2} = 6$, so the third triangular number is 6.

   (a) There is one positive integer a so that $T(4) + T(a) = T(10)$. Determine a.

   (b) Determine the smallest integer $b > 2011$ such that $T(b+1) - T(b) = T(x)$ for some positive integer x.

   (c) If $T(c) + T(d) = T(e)$ and $c + d + e = T(28)$, then show that $cd = 407(c + d - 203)$.

   (d) Determine all triples (c, d, e) of positive integers such that $T(c) + T(d) = T(e)$ and $c + d + e = T(28)$, where $c \leq d \leq e$.

2010 Galois Contest (Grade 10)

Friday, April 9, 2010

1. Emily's old showerhead used 18 L of water per minute.
   She installs a new showerhead that uses 13 L per minute.

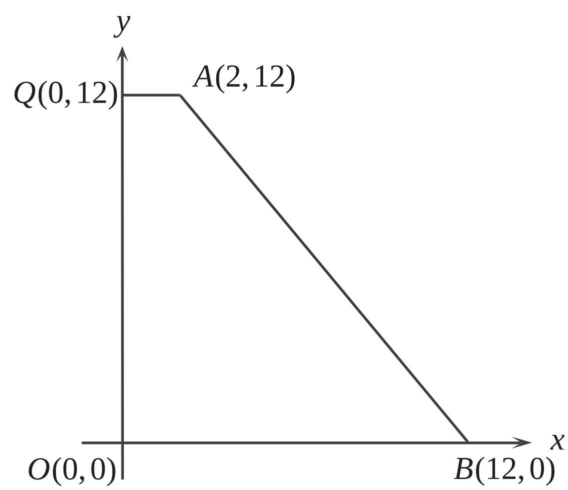
   (a) When Emily takes a bath, she uses 260 L of water. Using the new showerhead, what length of shower, in minutes, uses 260 L of water?

   (b) How much *less* water is used for a 10 minute shower with the new showerhead than with the old showerhead?

   (c) Emily is charged 8 cents per 100 L of water that she uses. Using the new showerhead instead of the old showerhead saves water and so saves Emily money. How much money does Emily *save* in water costs for a 15 minute shower?

   (d) How many minutes of showering, using the new showerhead, will it take for Emily to have saved $30 in water costs?

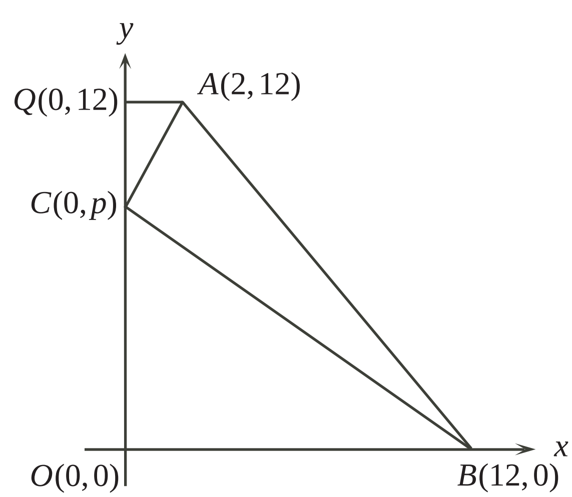
2. (a) Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.



   (b) Point $C(0,p)$ lies on the y-axis between $Q(0,12)$ and $O(0,0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of p.

   (c) Determine an expression for the area of $\triangle QCA$ in terms of p.

   (d) If the area of $\triangle ABC$ is 27, determine the value of p.



3. (a) Solve the system of equations algebraically for (x, y):

$$\begin{aligned} x + y &= 42 \\ x - y &= 10 \end{aligned}$$

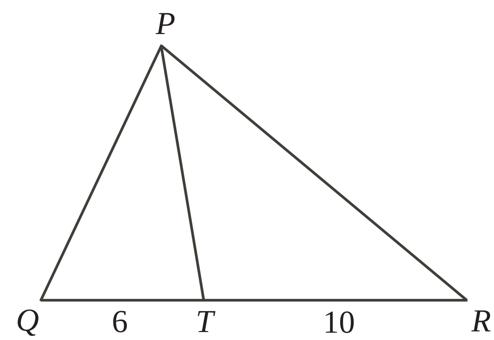
(b) Suppose that p is an even integer and that q is an odd integer. Explain why the system of equations

$$\begin{aligned} x + y &= p \\ x - y &= q \end{aligned}$$

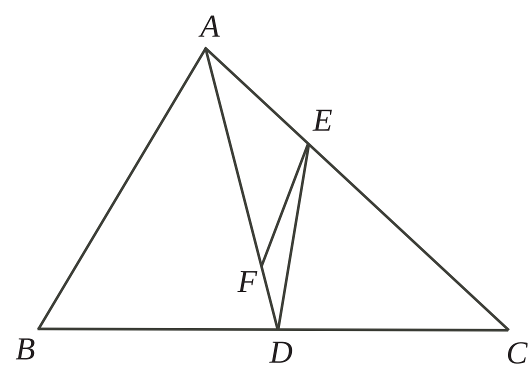
has no positive integer solutions (x, y).

(c) Determine all pairs of positive integers (x, y) that satisfy the equation $x^2 - y^2 = 420$.

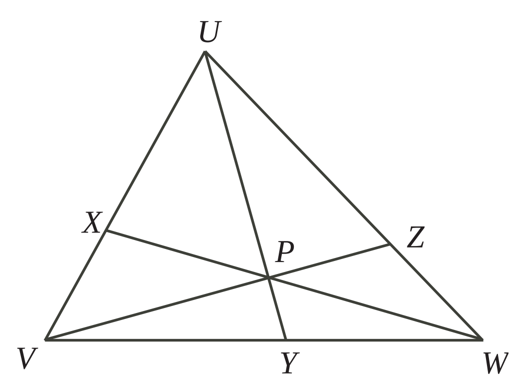
4. (a) In $\triangle PQR$, point T is on side QR such that $QT = 6$ and $TR = 10$. Explain why the ratio of the area of $\triangle PQT$ to the area of $\triangle PTR$ is $3 : 5$.



(b) In $\triangle ABC$, point D is the midpoint of side BC. Point E is on AC such that $AE : EC = 1 : 2$. Point F is on AD such that $AF : FD = 3 : 1$. If the area of $\triangle DEF$ is 17, determine the area of $\triangle ABC$.



(c) In the diagram, points X, Y and Z are on the sides of $\triangle UVW$, as shown. Line segments UY, VZ and WX intersect at P. Point Y is on VW such that $VY : YW = 4 : 3$. If $\triangle PYW$ has an area of 30 and $\triangle PZW$ has an area of 35, determine the area of $\triangle UXP$.



2009 Galois Contest (Grade 10)

Wednesday, April 8, 2009

1. Alex counts the number of students in her class with each hair colour, and summarizes the results in the following table:

Hair Colour	Number of Students
Blonde	8
Brown	7
Red	3
Black	2

   (a) What percentage of students in the class have blonde hair?

   (b) What percentage of students in the class have red or black hair?

   (c) How many students in the class with blonde hair would have to dye their hair black for the percentage of students in the class with black hair to be 20%?

   (d) How many students with red hair would have to join the class so the percentage of students in the class with red hair is equal to 32%?

2. A square has vertices with coordinates $A(6,9)$, $B(12,6)$, $C(t,0)$, and $D(3,3)$.

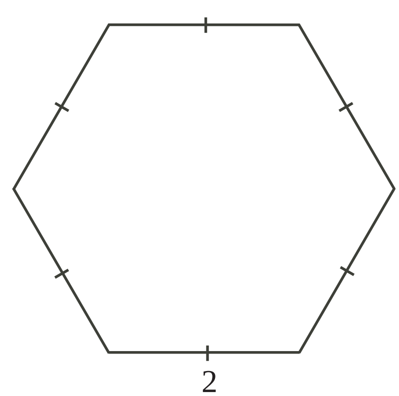
   (a) Determine the value of t, the x-coordinate of vertex C.

   (b) A line is drawn through $O(0,0)$ and D. This line meets AB at E. Determine the coordinates of E.

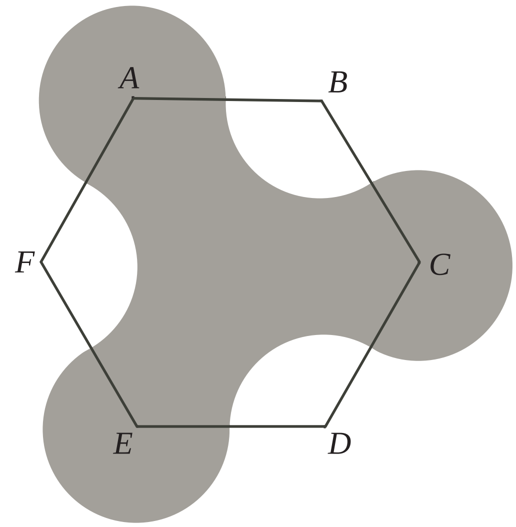
   (c) Determine the perimeter of quadrilateral $EBCD$.

3. (a) Find the area of an equilateral triangle with side length 2.

   (b) Determine the area of a regular hexagon with side length 2.



(c) In the diagram, regular hexagon $ABCDEF$ has sides of length 2. Using A, C and E as centres, portions of circles with radius 1 are drawn outside the hexagon. Using B, D and F as centres, portions of circles with radius 1 are drawn inside the hexagon. These six circular arcs join together to form a curve. Determine the area of the shaded region enclosed by this curve.



4. If m is a positive integer, the symbol $m!$ is used to represent the product of the integers from 1 to m. That is, $m! = m(m-1)(m-2)\cdots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$.
Some positive integers n can be written in the form

$$n = a(1!) + b(2!) + c(3!) + d(4!) + e(5!).$$

In addition, each of the following conditions is satisfied:

- a, b, c, d, and e are integers
- $0 \leq a \leq 1$
- $0 \leq b \leq 2$
- $0 \leq c \leq 3$
- $0 \leq d \leq 4$
- $0 \leq e \leq 5$.

(a) Determine the largest positive integer N that can be written in this form.

(b) Write $n = 653$ in this form.

(c) Prove that all integers n, where $0 \leq n \leq N$, can be written in this form.

(d) Determine the sum of all integers n that can be written in this form with $c = 0$.

2008 Galois Contest (Grade 10)

Wednesday, April 16, 2008

1. Three positive integers a, b and x form an O'Hara triple (a, b, x) if $\sqrt{a} + \sqrt{b} = x$. For example, $(1, 4, 3)$ is an O'Hara triple because $\sqrt{1} + \sqrt{4} = 3$.

   (a) If $(36, 25, x)$ is an O'Hara triple, determine the value of x.

   (b) If $(a, 9, 5)$ is an O'Hara triple, determine the value of a.

   (c) Determine the five O'Hara triples with $x = 6$. Explain how you found these triples.

2. (a) Determine the equation of the line passing through the points $P(0, 5)$ and $Q(6, 9)$.

   (b) A line, through Q, is perpendicular to PQ. Determine the equation of the line.

   (c) The line from (b) crosses the x-axis at R. Determine the coordinates of R.

   (d) Determine the area of right-angled $\triangle PQR$.

3. (a) A class of 20 students was given a two question quiz. The results are listed below:

Question number	Number of students who answered correctly
1	18
2	14

   Determine the smallest possible number and the largest possible number of students that could have answered both questions correctly. Explain why these are the smallest and largest possible numbers.

   (b) A class of 20 students was given a three question quiz. The results are listed below:

Question number	Number of students who answered correctly
1	18
2	14
3	12

   Determine the smallest possible number and the largest possible number of students that could have answered all three questions correctly. Explain why these are the smallest and largest possible numbers.

   (c) A class of 20 students was given a three question quiz. The results are listed below:

Question number	Number of students who answered correctly
1	x
2	y
3	z

   where $x \geq y \geq z$ and $x + y + z \geq 40$.
   Determine the smallest possible number of students who could have answered all three questions correctly in terms of x, y and z.

4. Carolyn and Paul are playing a game starting with a list of the integers 1 to n. The rules of the game are:

- Carolyn always has the first turn.
- Carolyn and Paul alternate turns.
- On each of her turns, Carolyn must remove one number from the list such that this number has at least one positive divisor other than itself remaining in the list.
- On each of his turns, Paul must remove from the list all of the positive divisors of the number that Carolyn has just removed.
- If Carolyn cannot remove any more numbers, then Paul removes the rest of the numbers.

For example, if $n = 6$, a possible sequence of moves is shown in this chart:

Player	Number(s) removed	Number(s) remaining	Notes
Carolyn	4	1, 2, 3, 5, 6	
Paul	1, 2	3, 5, 6	
Carolyn	6	3, 5	She could not remove 3 or 5
Paul	3	5	
Carolyn	None	5	Carolyn cannot remove any number
Paul	5	None	

In this example, the sum of the numbers removed by Carolyn is $4 + 6 = 10$ and the sum of the numbers removed by Paul is $1 + 2 + 3 + 5 = 11$.

(a) Suppose that $n = 6$ and Carolyn removes the integer 2 on her first turn. Determine the sum of the numbers that Carolyn removes and the sum of the numbers that Paul removes.

(b) If $n = 10$, determine Carolyn's maximum possible final sum. Prove that this sum is her maximum possible sum.

(c) If $n = 14$, prove that Carolyn cannot remove 7 numbers.

2007 Galois Contest (Grade 10)
Wednesday, April 18, 2007

1. Jim shops at a strange fruit store. Instead of putting prices on each item, the mathematical store owner will answer questions about combinations of items.

   (a) In Aisle 1, Jim receives the following answers to his questions:

Jim's Question	Answer
What is the sum of the prices of an Apple and a Cherry?	62 cents
What is the sum of the prices of a Banana and a Cherry?	66 cents

   What is difference between the prices of an Apple and a Banana? Which has a higher price? Explain how you obtained your answer.

   (b) In Aisle 2, Jim receives the following answers to his questions:

Jim's Question	Answer
What is the sum of the prices of a Mango and a Nectarine?	60 cents
What is the sum of the prices of a Pear and a Nectarine?	60 cents
What is the sum of the prices of a Mango and a Pear?	68 cents

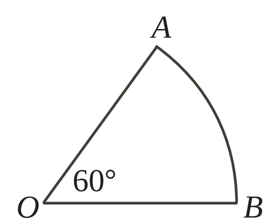
   What is the price of a Pear? Explain how you obtained your answer.

   (c) In Aisle 3, Jim receives the following answers to his questions:

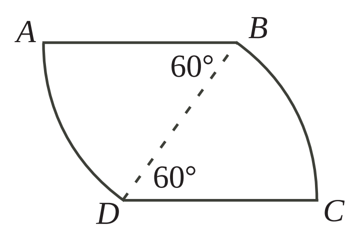
Jim's Question	Answer
What is the sum of the prices of a Tangerine and a Lemon?	60 cents
How much more does a Tangerine cost than a Grapefruit?	6 cents
What is the sum of the prices of Grapefruit, a Tangerine and a Lemon?	94 cents

   What is the price of a Lemon? Explain how you obtained your answer.

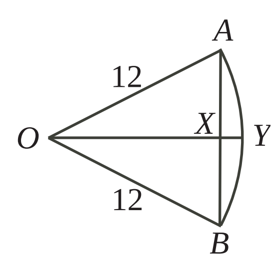
2. (a) In the diagram, what is the perimeter of the sector of the circle with radius 12? Explain how you obtained your answer.



   (b) Two sectors of a circle of radius 12 are placed side by side, as shown. Determine the *area* of figure $ABCD$. Explain how you obtained your answer.

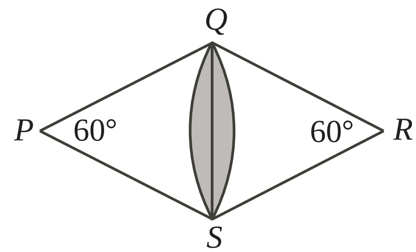


   (c) In the diagram, AOB is a sector of a circle with $\angle AOB = 60°$. OY is drawn perpendicular to AB and intersects AB at X. What is the length of XY? Explain how you obtained your answer.

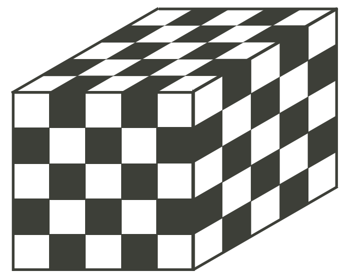


   (d) See over...

(d) Two sectors of a circle of radius 12 overlap as shown. Determine the area of the shaded region. Explain how you obtained your answer.



3. (a) Each face of a 5 by 5 by 5 wooden cube is divided into 1 by 1 squares. Each square is painted black or white, as shown. Next, the cube is cut into 1 by 1 by 1 cubes. How many of these cubes have *at least* two painted faces? Explain how you obtained your answer.



(b) A $(2k+1)$ by $(2k+1)$ by $(2k+1)$ cube, where k is a positive integer, is painted in the same manner as the 5 by 5 by 5 cube with white squares in the corners. Again, the cube is cut into 1 by 1 by 1 cubes.

i. In terms of k, how many of these cubes have *exactly* two white faces? Explain how you obtained your answer.

ii. Prove that there is no value of k for which the number of cubes having *at least* two white faces is 2006.

4. Jill has a container of small cylindrical rods in six different colours. Each colour of rod has a different length as summarized in the chart.

Colour	Length
Green	3 cm
Pink	4 cm
Yellow	5 cm
Black	7 cm
Violet	8 cm
Red	9 cm

These rods can be attached together to form a pole.
There are 2 ways to choose a set of yellow and green rods that will form a pole 29 cm in length: 8 green rods and 1 yellow rod OR 3 green rods and 4 yellow rods.

(a) How many different sets of yellow and green rods can be chosen that will form a pole 62 cm long? Explain how you obtained your answer.

(b) Among the green, yellow, black and red rods, find, with justification, two colours for which it is impossible to make a pole 62 cm in length using only rods of those two colours.

(c) If at least 81 rods of each of the colours green, pink, violet, and red must be used, how many different sets of rods of these four colours can be chosen that will form a pole 2007 cm in length? Explain how you got your answer.

2006 Galois Contest (Grade 10)

Thursday, April 20, 2006

1. A hat contains six slips of paper numbered from 1 to 6. Amelie and Bob each choose three slips from the hat without replacing any of the slips. Each of them adds up the numbers on his slips.

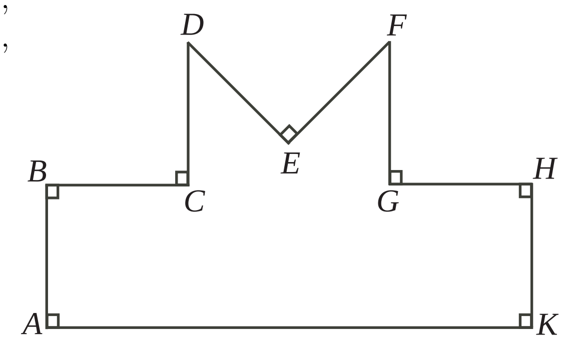
   (a) Determine the largest possible difference between Amelie's total and Bob's total. Explain how you found this difference.

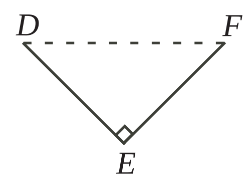
   (b) List all possible groups of three slips that Amelie can choose so that her total is one more than Bob's total.

   (c) Explain why it is impossible for Amelie and Bob to have the same total no matter which three slips each chooses.

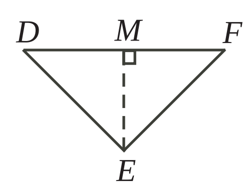
   (d) If more slips of paper are added to the hat, numbered consecutively from 7 to n, what is the smallest value of $n > 6$ so that Amelie and Bob can each choose half of the slips numbered from 1 to n and obtain the same total? Explain why this value of n works.

2. In the diagram, AB, BC, CD, DE, EF, FG, GH, and HK all have length 4, and all angles are right angles, with the exception of the angles at D and F.





   (a) Determine the length of DF.

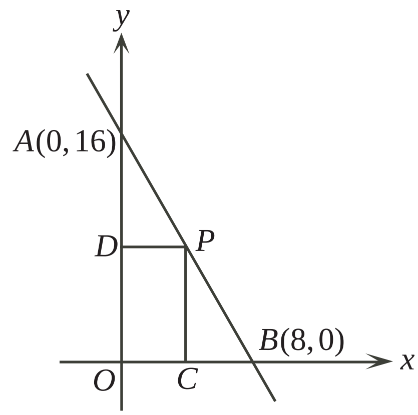


   (b) If perpendicular EM is drawn from E to DF, what is the length of EM? Explain how you got your answer.

   (c) If perpendicular EP is drawn from E to AK, what is the length of EP? Explain how you got your answer.

   (d) What is the area of figure $ABCDEFGHK$? Explain how you got your answer.

3. In the diagram, a line is drawn through the points $A(0,16)$ and $B(8,0)$. Point P is chosen in the first quadrant on the line through A and B. Points C and D are then chosen on the x-axis and y-axis, respectively, so that $PDOC$ is a rectangle.



   (a) Determine the equation of the line through A and B.

   (b) Determine the coordinates of the point P so that $PDOC$ is a square.

   (c) Determine the coordinates of all points P that can be chosen so that the area of rectangle $PDOC$ is 30.

4. (a) When the number 14 has its digits reversed to form the number 41, it is increased by 27. Determine all 2-digit numbers which are increased by 27 when their digits are reversed.

   (b) Choose any three-digit integer $\underline{a}\,\underline{b}\,\underline{c}$ whose digits are all different.
   (When a three-digit integer is written in terms of its digits as $\underline{a}\,\underline{b}\,\underline{c}$, it means the integer is $100a + 10b + c$.)
   Reverse the order of the digits to get a new three-digit integer $\underline{c}\,\underline{b}\,\underline{a}$.
   Subtract the smaller of these integers from the larger to obtain a three-digit integer $\underline{r}\,\underline{s}\,\underline{t}$, where r is allowed to be 0.
   Reverse the order of the digits of this integer to get the integer $\underline{t}\,\underline{s}\,\underline{r}$.
   Prove that, no matter what three-digit integer $\underline{a}\,\underline{b}\,\underline{c}$ you start with, $\underline{r}\,\underline{s}\,\underline{t} + \underline{t}\,\underline{s}\,\underline{r} = 1089$.

   (c) Suppose that $N = \underline{a}\,\underline{b}\,\underline{c}\,\underline{d}$ is a four-digit integer with $a \leq b \leq c \leq d$.
   When the order of the digits of N is reversed to form the integer M, N is increased by P. (Again, the first digit of P is allowed to be 0.)
   When the order of the digits of P is reversed, an integer Q is formed.
   Determine, with justification, all possible values of $P + Q$.

2005 Galois Contest (Grade 10)
Wednesday, April 20, 2005

1. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant.
   For example, the sequence 2, 11, 20, 29, . . . is an arithmetic sequence.
   (The ". . ." indicates that this sequence continues without ever ending.)

   (a) Find the 11th term in the arithmetic sequence 17, 22, 27, 32, ...

   (b) Explain why there is no number which occurs in each of the following arithmetic sequences:

   17, 22, 27, 32, . . .
   13, 28, 43, 58, . . .

   (c) Find a number between 400 and 420 which occurs in both of the following arithmetic sequences:

   17, 22, 27, 32, . . .
   16, 22, 28, 34, . . .

   Explain how you got your answer.

2. Emilia and Omar are playing a game in which they take turns placing numbered tiles on the grid shown.

Emilia starts the game with six tiles: $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, and $\boxed{6}$.

Omar also starts the game with six tiles: $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, and $\boxed{6}$.

Once a tile is placed, it cannot be moved.
After all of the tiles have been placed, Emilia scores one point for each row that has an even sum and one point for each column that has an even sum. Omar scores one point for each row that has an odd sum and one point for each column that has an odd sum. For example, if the game ends with the tiles placed as shown below, then Emilia will score 5 points and Omar 2 points.

3	1	2	4
5	5	2	4
1	3	6	6

(a) In a game, after Omar has placed his second last tile, the grid appears as shown below. Starting with the partially completed game shown, give a final placement of tiles for which Omar scores more points than Emilia. (You do not have to give a strategy, simply fill in the final grid.)

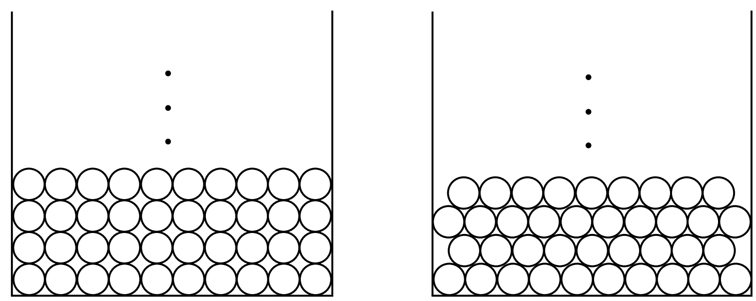
1	3		5
3	1	2	2
	4	4	

(b) Explain why it is impossible for Omar and Emilia to score the same number of points in any game.

(c) In the partially completed game shown below, it is Omar's turn to play and he has a 2 and a 5 still to place. Explain why Omar cannot score more points than Emilia, no matter where he places the 5.

1		3	6
	5		4
3		1	6

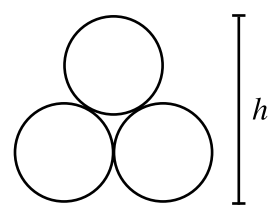
3. Two identical rectangular crates are packed with cylindrical pipes, using different methods. Each pipe has diameter 10 cm. A side view of the first four rows of each of the two different methods of packing is shown below.



Crate A Crate B

(a) If 200 pipes are packed in each of the two crates, how many rows of pipes are there in each crate? Explain your answer.

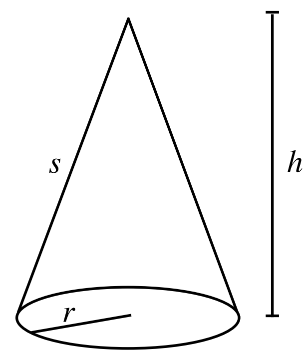
(b) Three pipes from Crate B are shown. Determine the height, h, of this pile of 3 pipes. Explain your answer.



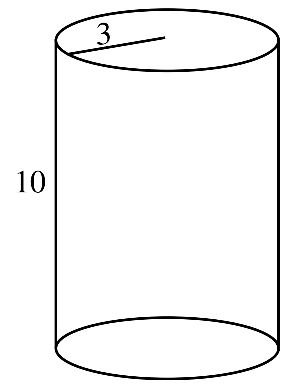
(c) After the crates have been packed with 200 pipes each, what is the difference in the total heights of the two packings? Explain your answer.

4. The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

   The *total* surface area of a cone with height h, slant height s, and radius r is $\pi r^2 + \pi rs$.



   (a) A cylinder has a height of 10 and a radius of 3. Determine the *total* surface area, including the two ends, of the cylinder, and also determine the volume of the cylinder.



   (b) A cone, a cylinder and a sphere all have radius r. The height of the cylinder is H and the height of the cone is h. The cylinder and the sphere have the same volume. The cone and the cylinder have the same total surface area. Prove that h and H cannot both be integers.

2004 Galois Contest (*Grade 10*)

Thursday, April 15, 2004

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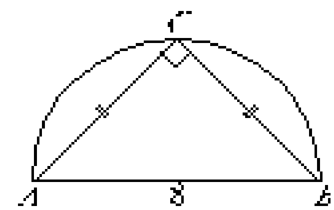
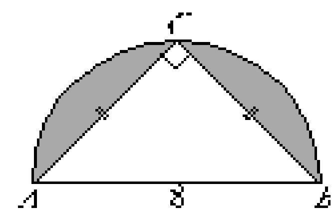
1. The Galois Group is giving out four types of prizes, valued at \$5, \$25, \$125 and \$625.
   (a) The Group gives out at least one of each type of prize. If five prizes are given out with a total value of \$905, how many of each type of prize is given out? Explain how you got your answer.
   (b) If the Group gives out at least one of each type of prize and five prizes in total, determine the other three possible total values it can give out. Explain how you got your answer.
   (c) There are two ways in which the Group could give away prizes totalling \$880 while making sure to give away at least one and at most six of each prize. Determine the two ways of doing this, and explain how you got your answer.

2. In the diagram, the semicircle has diameter $AB = 8$. Point C is on the semicircle so that triangle ABC is isosceles and right-angled.

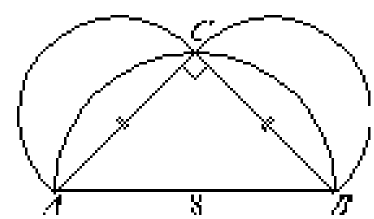


   (a) Determine the area of triangle ABC.

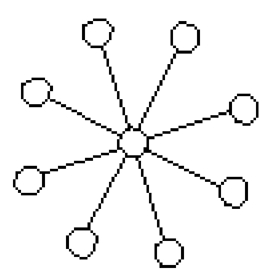
   (b) The two regions inside the semicircle but outside the triangle are shaded. Determine the total area of the two shaded regions.



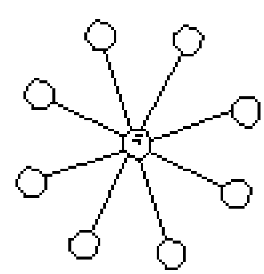
   (c) Semicircles are drawn on AC and CB, as shown. Show that:
   (Area of semicircle drawn on AB)
   = (Area of semicircle drawn on AC) + (Area of semicircle drawn on CB)



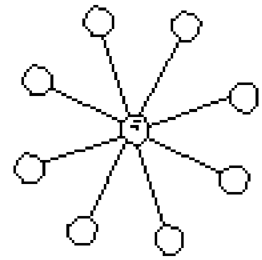
3. In "The Sun Game", two players take turns placing discs numbered 1 to 9 in the circles on the board. Each number may only be used once. The object of the game is to be the first to place a disc so that the sum of the 3 numbers along a line through the centre circle is 15.



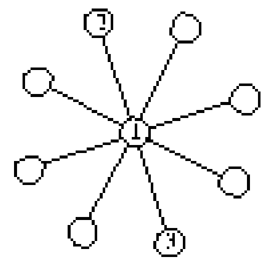
   (a) If Avril places a 5 in the centre circle and then Bob places a 3, explain how Avril can win on her next turn.



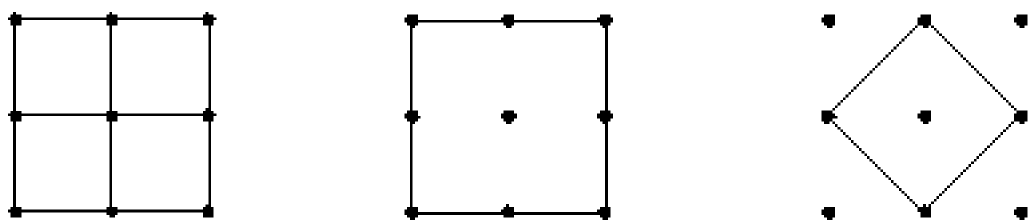
(b) If Avril starts by placing a 5 in the centre circle, show that whatever Bob does on his first turn, Avril can always win on her next turn.



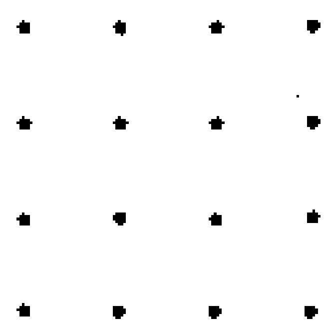
(c) If the game is in the position shown and Bob goes next, show that however Bob plays, Avril can win this game.



4. A 3 by 3 grid has dots spaced 1 unit apart both horizontally and vertically. Six squares of various side lengths can be formed with corners on the dots, as shown.



(a) Given a similar 4 by 4 grid of dots, there is a total of 20 squares of five different sizes that can be formed with corners on the dots. Draw one example of each size and indicate the number of squares there are of that size.



(b) In a 10 by 10 grid of dots, the number of squares that can be formed with side length $\sqrt{29}$ is two times the number of squares that can be formed with side length 7. Explain why this is true.

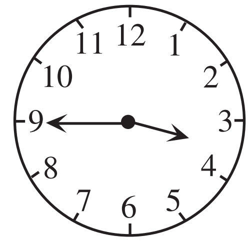
(c) Show that the total number of squares that can be formed in a 10 by 10 grid is
$1(9^2)+2(8^2)+3(7^2)+4(6^2)+5(5^2)+6(4^2)+7(3^2)+8(2^2)+9(1^2)$.

2003 Galois Contest *(Grade 10)*

Wednesday, April 16, 2003

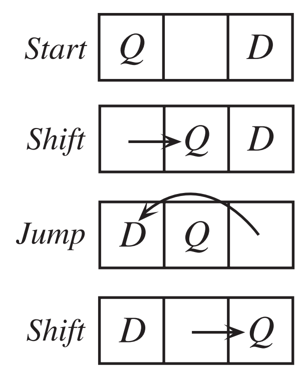
1. (a) The *sum of the squares* of 5 consecutive positive integers is 1815. What is the largest of these integers?
   (b) Show that the sum of the squares of any 5 consecutive integers is divisible by 5.

2. Professor Cuckoo mistakenly thinks that the angle between the minute hand and the hour hand of a clock at 3:45 is 180°.
   (a) Through how many degrees does the hour hand pass as the time changes from 3:00 p.m. to 3:45 p.m.?
   (b) Show that the Professor is wrong by determining the exact angle between the hands of a clock at 3:45.
   (c) At what time between 3:00 and 4:00 will the angle between the hands be 180°?



3. In the game "Switch", the goal is to make the dimes (D) and quarters (Q) switch spots. The starting position of the game with 1 quarter and 1 dime is shown below. Allowable moves are:
   (i) If there is a vacant spot beside a coin then you may *shift* to that space.
   (ii) You may *jump* a quarter with a dime or a dime with a quarter if the space on the other side is free.

   The game shown in the diagram takes three moves.



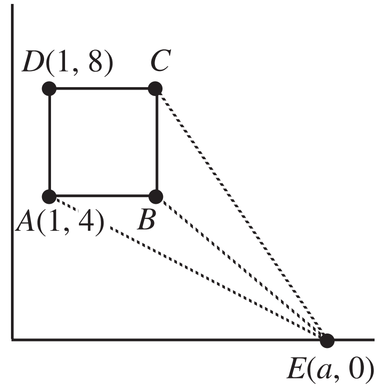
   (a) Complete the diagram to demonstrate how the game of "Switch" that starts with 2 quarters and 2 dimes can be played in 8 moves.

Q	*Q*		*D*	*D*
D	*D*		*Q*	*Q*

   (b) By considering the number of required *shifts* and *jumps*, explain why the game with 3 quarters and 3 dimes cannot be played in fewer than 15 moves.

over ...

4. In the diagram, $ABCD$ is a square and the coordinates of A and D are as shown.
   (a) The point $E(a, 0)$ is on the x-axis so that the triangles CBE and ABE lie entirely outside the square $ABCD$. For what value of a is the sum of the areas of triangles CBE and ABE equal to the area of square $ABCD$?
   (b) The point F is on the line passing through the points $M(6, -1)$ and $N(12, 2)$ so that the triangles CBF and ABF lie entirely outside the square $ABCD$. Determine the coordinates of the point F if the sum of the areas of triangle CBF and ABF equals the area of square $ABCD$.



Extensions (Attempt these only when you have completed as much as possible of the four main problems.)

Extension to Problem 1:
The number 1815 is also the sum of 5 consecutive positive integers. Find the next number larger than 1815 which is the sum of 5 consecutive integers and also the sum of the squares of 5 consecutive integers.

Extension to Problem 2:
The assumption might be made that there are 24 times during any 12 hour period when the angle between the hour hand and the minute hand is 90°. This is not the case. Determine the actual number of times that the angle between the hour and minute hands is 90°.

Extension to Problem 3:
Explain why the game with n quarters and n dimes cannot be played in fewer than $n(n+2)$ moves.

Extension to Problem 4:
Find the set of all points $P(x, y)$ which satisfy the conditions that the triangles CBP and ABP lie entirely outside the square $ABCD$ and the sum of the areas of triangles CBP and ABP equals the area of square $ABCD$.