The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Galois Contest

(Grade 10)

Thursday, April 4, 2024
(in North America and South America)

Friday, April 5, 2024
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

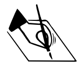
1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

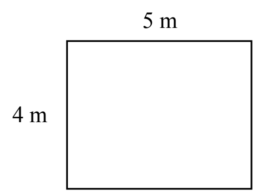
- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

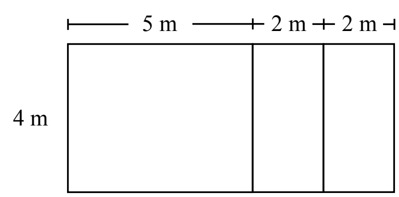
The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

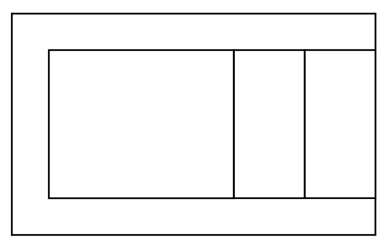
1. Three students are helping to expand their school's garden. Initially, the garden has a length of 5 m and a width of 4 m, as shown.



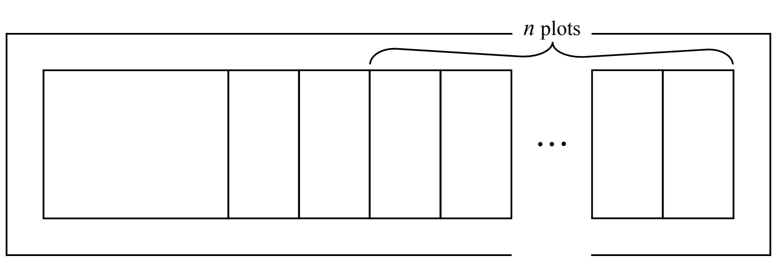
 (a) Rob adds two additional 2 m by 4 m plots side by side next to the initial garden, as shown. What is the total area of the expanded garden after Rob adds these two plots?



 (b) Kirima adds a path around three sides of the previous garden, as shown. If the width of the path is 1 m, what is the total combined area of the garden and the path?



 (c) Noah adds n additional 2 m by 4 m plots to the previous version of the garden (in part (b)), and then continues the 1 m wide path so that it surrounds the entire garden, as shown. If the total combined area of the garden and the path is 150 m^2, determine the value of n.



2. When a point (x, y) is rotated $90°$ clockwise about the origin, the resulting coordinates are $(y, -x)$. We call this rotation R. When a point (x, y) is translated up 2 units, the resulting coordinates are $(x, y + 2)$. We call this translation T. For example, beginning with the point $(8, -2)$, and applying R then T, the resulting coordinates are $(-2, -6)$, as shown:

$$(8, -2) \xrightarrow{R} (-2, -8) \xrightarrow{T} (-2, -6)$$

(a) Beginning with the point $(5, 11)$, and applying R then T, what are the resulting coordinates?

(b) Beginning with the point $(-3, 7)$, when R is applied 5 times, what are the resulting coordinates?

(c) Consider the following sequence of transformations: R, then R again, and then T. Beginning with the point $(9, 1)$, this sequence R, R, T is repeated a total of 11 times. Determine the resulting coordinates.

3. Seven black balls numbered 1, 2, 3, 4, 5, 6, and 7, are placed in a hat. Balls are drawn randomly one at a time from the hat. When a ball is drawn, it is neither replaced by another ball nor returned to the hat.

(a) What is the probability that the first ball drawn is even-numbered?

(b) What is the probability that the sum of the numbers on the first two balls drawn is equal to 5?

(c) Determine the probability that the sum of the numbers on the first two balls drawn is greater than or equal to 6.

(d) An eighth ball is added to the hat. This eighth ball is gold and it is numbered with an integer k, where $1 \leq k \leq 7$. The probability that the sum of the numbers on the first two balls drawn is greater than or equal to 7 is $\frac{3}{4}$. Determine the value of k.

4. In a $3 \times n$ rectangular grid, two cells are *neighbours* if they share an edge. A $3 \times n$ *Griffin Grid* is a $3 \times n$ grid, with $n \geq 2$, having the following properties:

- each cell contains either -1 or 1, and
- the number in each cell is equal to the product of the numbers in all cells that are neighbours.

-1	1
-1	-1
-1	1

An example of a 3×2 Griffin Grid is shown.

(a) Fill in the empty cells of the 3×5 grid shown so that it is a Griffin Grid.

-1				
1				
-1				

(b) Determine the total number of 3×5 Griffin Grids.

(c) Let S be the sum of the numbers of $3 \times n$ Griffin Grids for $2 \leq n \leq 2024$. Determine the value of S.

2024 Galois Contest (English)

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For students...

Thank you for writing the 2024 Galois Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2024.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2024/2025 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
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- Find your school's contest results