The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Thursday, April 4, 2024
(in North America and South America)

Friday, April 5, 2024
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. In a sequence of integers, the 1st term is 3. Each new term is obtained by adding 6 to the previous term. In this sequence, the first four terms are $3, 9, 15, 21$.

   (a) What is the 5th term?

   (b) What is the average (mean) of the 4th, 5th and 6th terms?

   (c) What is the 20th term?

   (d) Determine the smallest term that is greater than 1000.

2. Each day, Ella loaded her truck with red shirts and blue shirts for delivery to two stores. At Store 1, she dropped off some shirts, and at Store 2 she dropped off all remaining shirts.

   (a) On Monday, she loaded 800 red shirts and 200 blue shirts. At Store 1, she dropped off only red shirts. At Store 2, 50% of the shirts dropped off were red. How many red shirts were dropped off at Store 1?

   (b) On Tuesday, she loaded $5x$ red shirts and $5x$ blue shirts. At Store 1, she dropped off 40% of the red shirts, and no blue shirts. What percentage of the shirts dropped off at Store 2 were blue?

   (c) On Wednesday, she loaded $3y$ red shirts and y blue shirts. At Store 1, she dropped off some red shirts and no blue shirts. Additionally, she picked up some green shirts. At Store 2, she dropped off all remaining shirts and the numbers of red, green, and blue shirts dropped off were equal. Of all the shirts that she dropped off on Wednesday, determine the percentage that were green.

3. A square slice of bread, $ABCD$, has dimensions 30×30. The slice of bread has crusts on three of its edges, as shown by the bolded sides AB, BC and CD in Figure 1. When the slice is cut into smaller pieces, the smaller pieces are called *fair* if

- each piece has the same area, and
- each piece has the same length of crust.

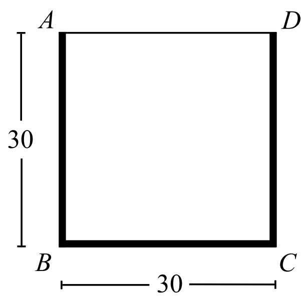


Figure 1

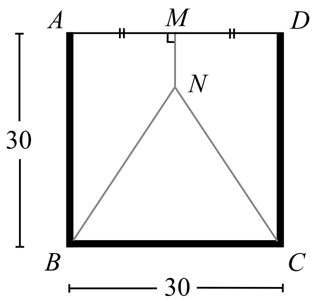


Figure 2

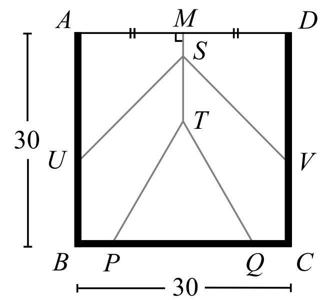


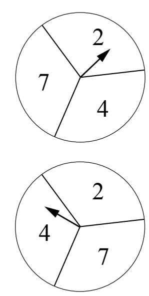
Figure 3

(a) In Figure 2, M is the midpoint of AD and MN is perpendicular to AD. If $ABCD$ is cut along the lines MN, NB and NC to create three fair pieces, what is the length of MN?

(b) In Figure 3, M is the midpoint of AD, MT is perpendicular to AD, and S lies on MT. If $ABCD$ is cut along the lines MT, TP, TQ, SU, and SV to create five fair pieces, what is the length of PQ?

(c) In Figure 3, determine the length of ST.

4. A game is played with spinners having three equally sized sections numbered with three distinct integers. Each game is played between two players. Each player spins their spinner once and the player who spins the highest number is the winner. Using either of the two spinners shown, a player could spin a 2, a 4 or a 7, each with a probability of $\frac{1}{3}$, and so the spinners would be considered to be the same and both are labelled $\{2, 4, 7\}$.



(a) Alice spins the spinner $\{5, 9, 11\}$ and Binh spins the spinner $\{1, 8, 10\}$. What is the probability that Alice wins?

(b) Carole makes the spinner $\{1, 5, 10\}$. Darsh makes a spinner by choosing three distinct integers from $2, 3, 4, 6, 7, 8, 9$. Determine how many different spinners Darsh can make so that his probability of winning is greater than Carole's probability of winning.

(c) Erin makes the spinner $\{5, 8, 15\}$ and Fynn makes the spinner $\{2, 10, 18\}$. Gina makes a spinner $\{x, y, z\}$ by choosing three integers $x < y < z$ from the list

$$1, 3, 4, 6, 7, 9, 11, 12, 13, 14, 16, 17, 19, 20$$

Suppose that

- the probability that Fynn beats Erin is equal to p,
- the probability that Erin beats Gina is equal to q, and
- the probability that Gina beats Fynn is equal to r.

Determine how many different spinners Gina can make so that $p = q = r$.

2024 Fryer Contest (English)

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For students...

Thank you for writing the 2024 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2024.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2024/2025 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
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